Lecture 19: Text Indexing I

COMP526: Efficient Algorithms

Updated: December 5, 2024

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Announcements

1. Quiz 07 on Error Correcting Codes
    - Complete by 11:59pm, Friday 06 November
2. Grading is slow (sorry)
    - Programming assignment 1 grades next week
3. Last lectures:
    - Text indexing (Today and next Tuesday)
    - Final review (next Thursday)
4. Attendance Code:

Meeting Goals

1. Introduce and analyze Parallel MergeSort
2. Introduce the text indexing problem
3. Define the trie data structure
4. Define suffix trees
5. Describe applications of suffix trees

Parallel
MergeSort

Last Time

Parallel Algorithms!

- PRAM model
    - Unlimited parallel processing elements (PEs)
- Brent's Theorem: span T and work W with unlimited PEs

  $\Longrightarrow$ span $O(T + W/p)$ and work $O(W)$ with p PEs
- Parallel string matching with span $T = O(m)$ and work $W = O(n)$
- Sorting networks
    - span $T = O(\log^2 n)$ and work $W = O(n \log^2 n)$
    - limited to specialized hardware and/or small arrays

Parallel Divide & Conquer?

Observation. The Divide & Conquer strategy can lend itself well to parallelism:

1. Divide problem into sub-tasks
2. Solve the subtasks

3. Merge solutions of the subtasks

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3. Merge solutions of the subtasks (**…?**)
    - How to parallelize this?

Parallel MergeSort?

Revisited: MERGESORT

```
procedure MERGESORT($A, i, k$)
    if $i < k$ then
        $j \leftarrow \lfloor (i+k)/2 \rfloor$
        MERGESORT($A, i, j$)
        MERGESORT($A, j+1, k$)
        $B \leftarrow$ COPY($A, i, j$)
        $C \leftarrow$ COPY($A, j+1, k$)
        MERGE($B, C, A, i$)
    end if
end procedure
```

Parallel MergeSort?

PollEverywhere

What is the span of MergeSort with parallel recursive calls and sequential merges?

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- For the other sub-array, use binary search!
- **Parallelize:** do each x in parallel!

Parallel MergeSort in Code

```
procedure ParallelMerge($A[l..m)$, $A[m..r)$, $B$)
    for $i = l, \ldots, m-1$ in parallel do
        $k \leftarrow (i-l) + \textsc{BinarySearch}(A[m..r), A[i])$
        $B[k] \leftarrow A[i]$
    end for
    for $j = m, m+1, \ldots, r-1$ in parallel do
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Improvements. Merge can be improved to $\Theta(n)$ work! (but it's complicated)

Concluding Thoughts

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Parallelism is Subtle

- Reasoning about parallel programs is hard
- Writing correct parallel programs is hard
- Idealized models abstract away many challenges
  - no perfect synchrony?
  - tolerate faults?

Text Indexing

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Previously: String Matching.

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- Focus on *one shot* complexity:
  - how long to search T for a single pattern P

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A Variation. The text T is *fixed*, but we may wish to search T for many different (initially) unknown patterns $P_1, P_2, \ldots$.

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- Applications:
  - web search engines
  - online dictionaries/encyclopedias
  - DNA/RNA databases
  - searching any collection of text documents

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An Alternative Approach. *Preprocess* the text T to make the searches more efficient

- Pay for preprocessing upfront
- Each query can be *much* more efficient.

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Example Problem. Given a text T of *words*, implement an *index* of the occurrences of that word.

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Question. How can we implement such a map *efficiently?*

The Trie Data Structure

Idea. Store words in a tree

- Each leaf represents a possible word in the text
- Each internal node represents *prefix* of a word in the text
    - path from root to leaf stores letters in the leaf word
- Append a terminating character to each word to make the tree a **prefix tree**

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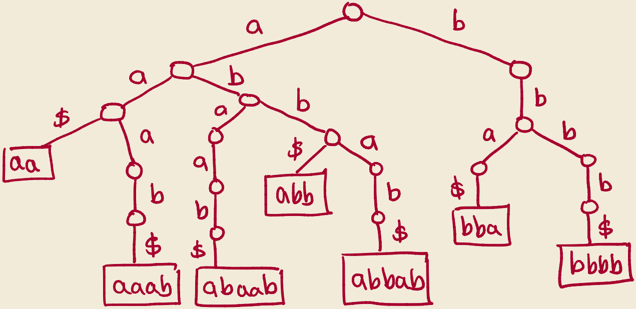
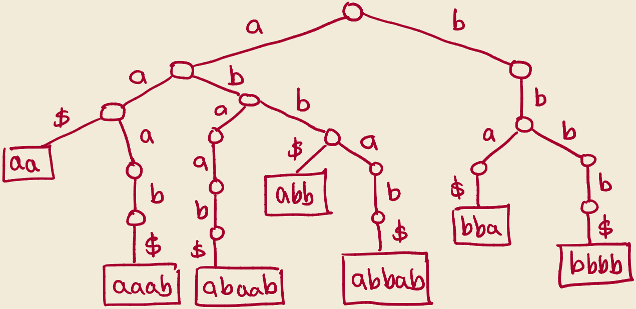
Example: {`aa$, aaab$, abaab$, abb$, abbab$, bba$, bbbb$`}



Searching a Trie

Question. Given a pattern P and a **trie** for the text T, how do we determine if T contains the pattern P?



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Procedure. Given the pattern $P[0..m)$:

1. Start at the root of the trie
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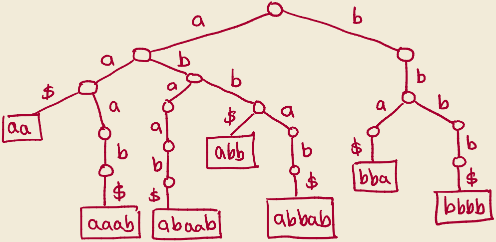

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PollEverywhere

What is the running time of searching a trie?

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Remarkable fact. The time to search a trie depends only on the length of P, not the size of T!

- Also: the trie can be computed efficiently from T (in $O(n)$ time).

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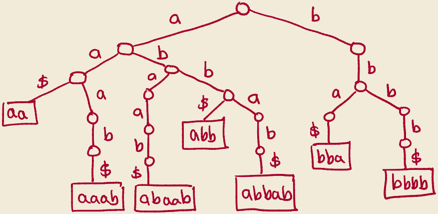
Idea. Compress paths without branches!

- Replace a path of **unary** (single-child) nodes with a single edge
- Label edge with the *first* character of the corresponding path
- Label each *vertex* with the index of the next character

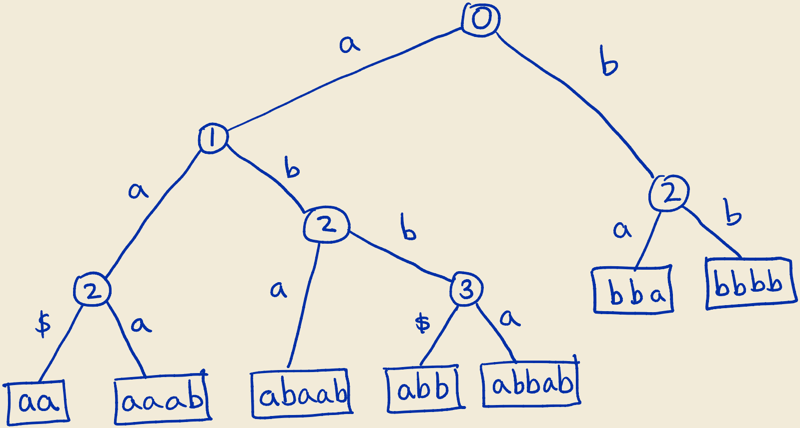
Words, Trie, Compact Trie

Example: {`aa$`, `aaab$`, `abaab$`, `abb$`, `abbab$`, `bba$`, `bbbb$`}



Compact Trie Features

Question. How do we search a *compact* trie?



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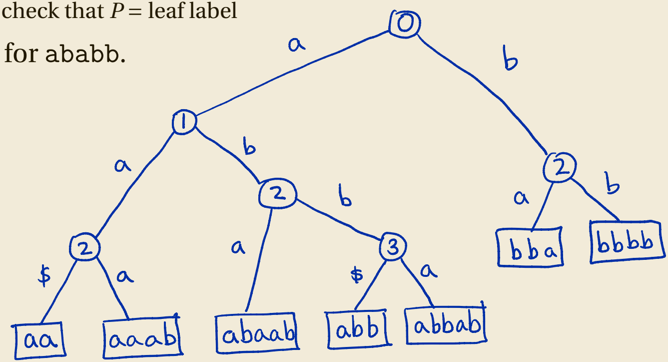
Example. Search for `ababb`.



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Useful feature. If a compact trie stores ℓ words, then it has at most $\ell - 1$ internal nodes as well.

- The size of the trie is proportional to the number of words it stores!
- **Fact** (to prove). If a tree T has ℓ leaves and every internal node has at least two children, then T has at most $2\ell - 1$ vertices.

Trie Discussion

Advantages of tries:

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We need new ideas!!

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An observation. $P[i, i+1)$, $P[i, i+2)$, $P[i, i+3), \ldots$ can all just be checked against $P[i, n)$

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Definition. Given a text $T[0..n)$ the **suffix tree** $\mathscr{T}$ of T is formed by:

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anana$
nana$
ana$
na$
a$
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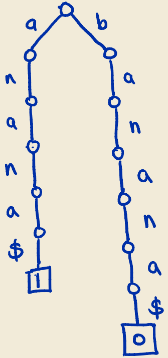

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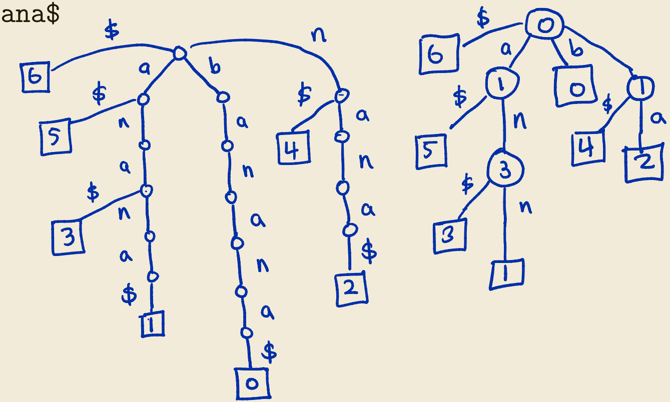

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PollEverywhere

Given $T[0..n)$, what is the total size of the associated suffix tree $\mathscr{T}$?

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For now. Take it as given that $\mathscr{T}$ can be computed in $O(n)$ time.

Suffix Tree Applications

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Application 1: String Matching

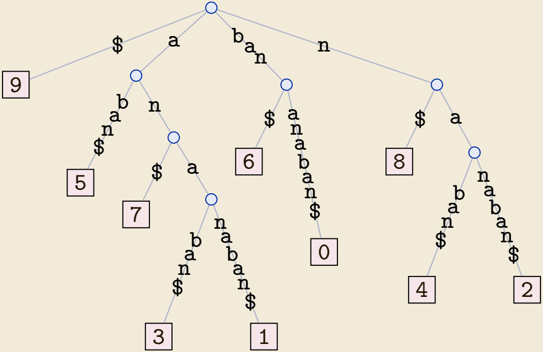
Observation. P occurs in $T \iff P$ is a prefix of a suffix of T.

- $\mathscr{T}$ stores (references to) all suffixes in T
- To search for P, try follow a path with label P until
  1. we get stuck
     - internal node without next character
     - mismatch along an edge
  2. we reach end of pattern P
     - all descendent leaves contain P!
  3. reach a leaf ℓ with part of P left (no match)

String Matching Example

Bananas. $T =$ `b a n a n a b a n $`

Human readible suffix tree:



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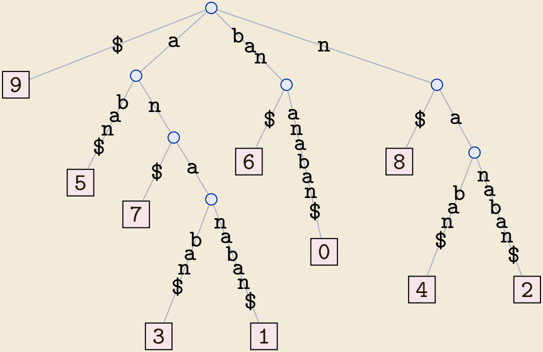

Note. Operations on “human readable” tree can be simulated in true suffix tree.

- each internal node stores pointer to left-most descendant index

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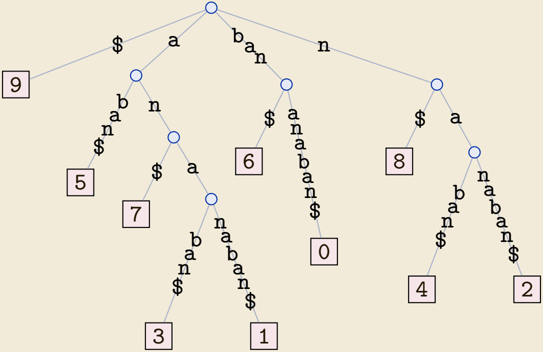

Search $P =$ ann

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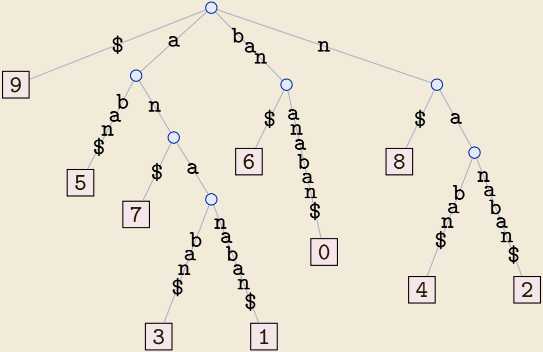

Search $P =$ `baa`

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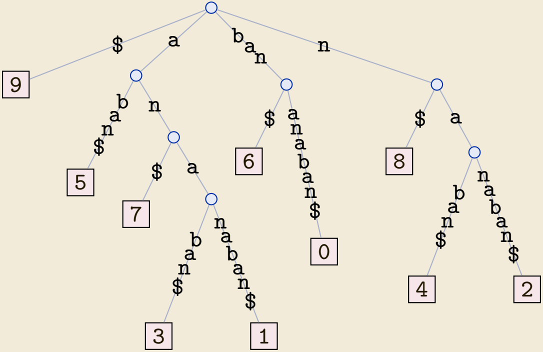

Search $P =$ `ana`

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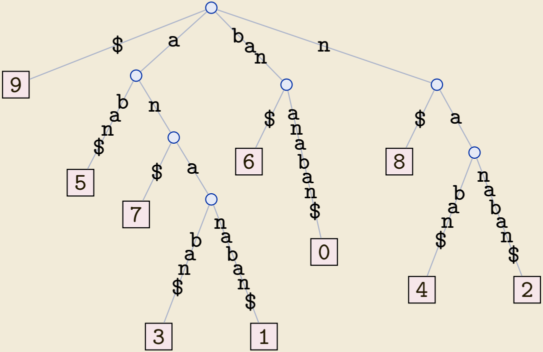

Search $P =$ `ba`

String Matching Discussion

Using Suffix Trees

- Pre-process a text $T[0..n)$ in $O(n)$ time *once*
- Search for $P[0..m)$ in time $O(m)$ time

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Comparison. If T is large and static, and we expect to perform many searches, the suffix tree construction is *much* more efficient!

Application 2: Repeated Substring

Problem. Given T, compute the **longest repeated substring** of T

- find the largest ℓ such that there are distinct indices i and j with $T[i, i+\ell] = T[j, j+\ell]$.

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Example. $T = \texttt{b a n a n a b a n}$

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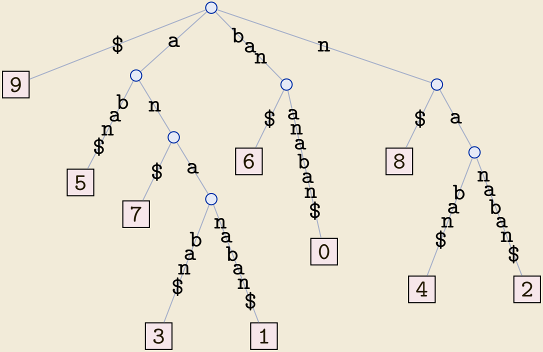
Repeated substrings in the suffix tree?



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Observation. Repeated substrings correspond to paths of internal nodes in $\mathscr{T}$.

- Longest repeated substring = longest path of internal nodes in $\mathscr{T}$
  - "longest path" includes weight for compressed edges
- Can be computed in $O(n)$ time!
  - use "depth first search" strategy

More Applications

Using suffix trees we can perform the following tasks efficiently:

1. **Longest Common Substring** in time $O(n_1 + n_2 + \cdots + n_k)$
    - Input: texts $T_1, T_2, \ldots, T_k$
    - Output: the longest substring that is contained in all T_i

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4. **Matching with Wildcards**
    - Input: text $T[0..n)$, pattern $P[0..m)$ with wildcards
        - wildcard character $*$ matches a substring of any length
    - Output: first appearance of P (with wildcard matches)

Conclusion

Suffix trees are amazing data structures!

- Tons of applications
- Surprising theoretical results

Next time. Constructing suffix trees *efficiently*

Scratch Notes