

# Lecture 21: Linearizability II

COSC 273: Parallel and Distributed  
Computing

Spring 2023

# Announcements

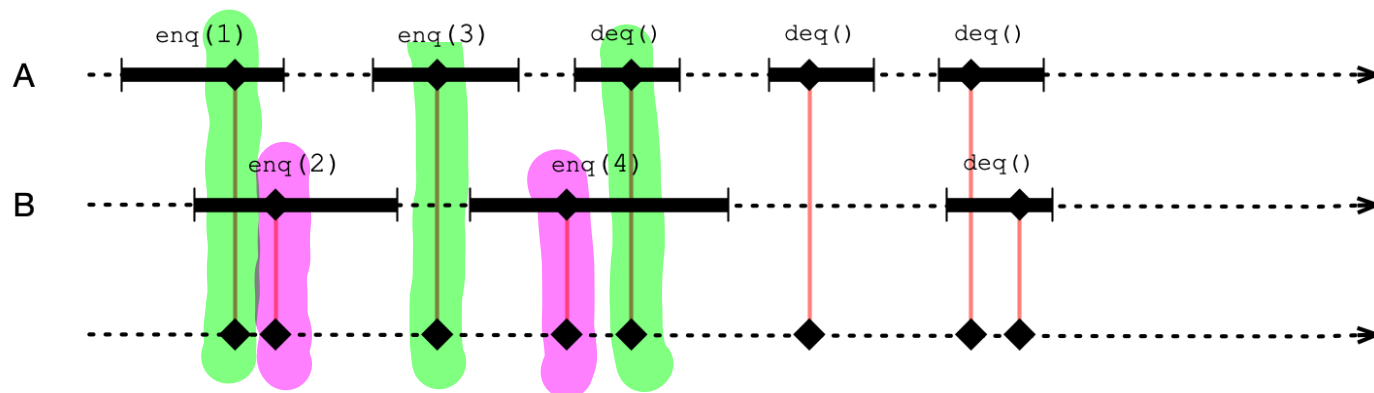
1. Quiz this Friday
  - sequential consistency
  - linearizability
  - recall **stack** operations
    - `push(x), pop()`

# Last Time: Linearizability

An execution of a shared object is **linearizable** if:

- exists a *linearization point* in each method call such that execution is consistent with sequential execution where method calls occur in order of corresponding linearization points

An implementation of an object is linearizable if every execution is linearizable.



# Linearizable TwoCounter

```
public class TwoCounter {  
    int[] counts = new int[2];  
    public void increment (int amt) {  
        • int i = ThreadID.get(); // thread IDs are 0 and 1  
        • int count = counts[i];  
        → counts[i] = count + amt;  
    }  
    public int read () {  
        → • int count = counts[0];  
        • count = count + counts[1];  
        • return count;  
    }  
}
```

0  
1

0

~~inc(3)~~

1

~~read()~~

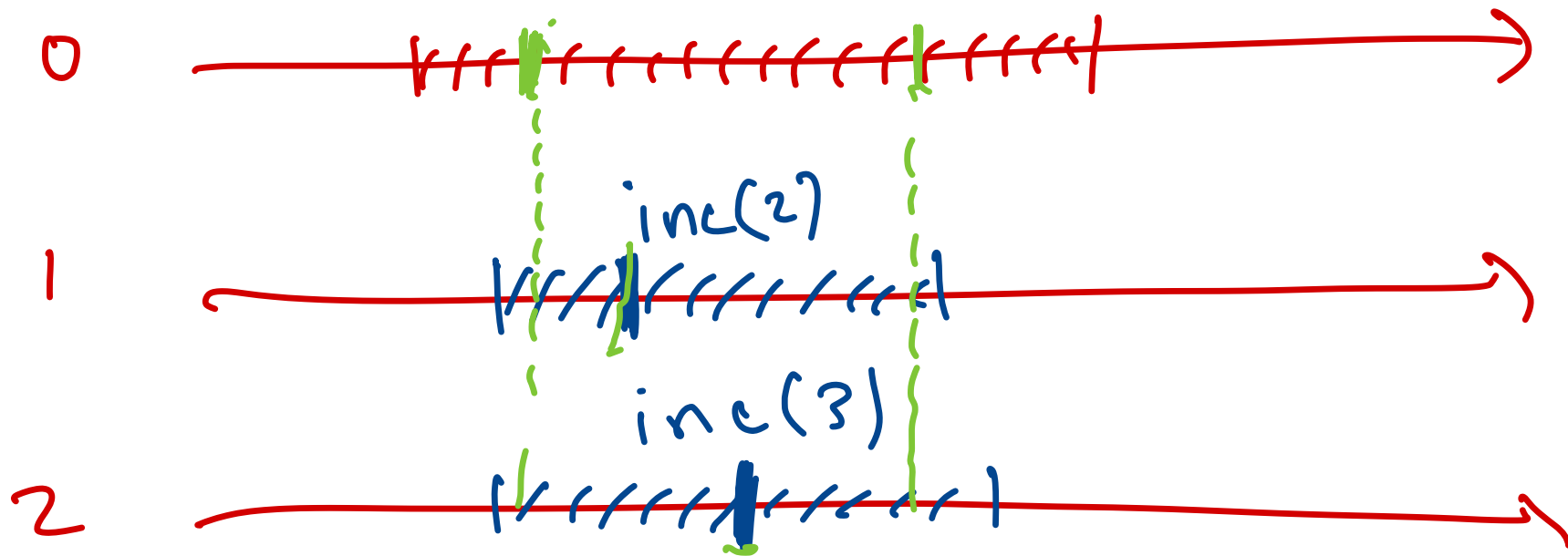
# ThreeCounter Example

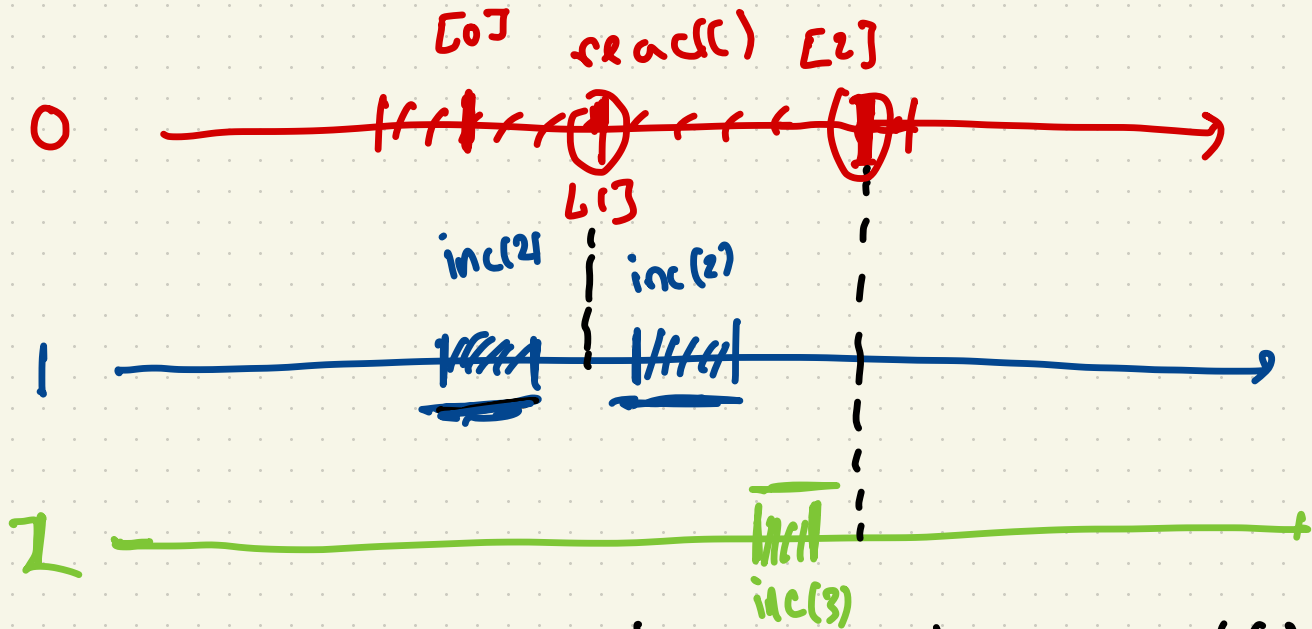
```
public class ThreeCounter {  
    int[] counts = new int[3];  
  
    public void increment (int amt) {  
        → int i = ThreadID.get(); // thread IDs are 0, 1, and 2  
        → int count = counts[i];  
        counts[i] = count + amt;  
    }  
}
```

# A read Method

```
public int read () {  
  (1) int count = counts[0];  
  (2) count = count + counts[1];  
  (3) count = count + counts[2];  
  return count;  
}
```

(2) read(3) ————— return(3)





What value is returned by `read()`?

What possible values from linearizable exec?

0, 2, 4, 7

Is ThreeCounter Linearizable?

Nope.



# Writing Between the Lines

```
public int read () {  
    int count = counts[0];  
    count = count + counts[1];  
    count = count + counts[2];  
    return count;  
}
```

# Sequentially Consistency

## Questions.

1. Is the previous execution sequentially consistent?
2. Is ThreeCounter sequentially consistent?

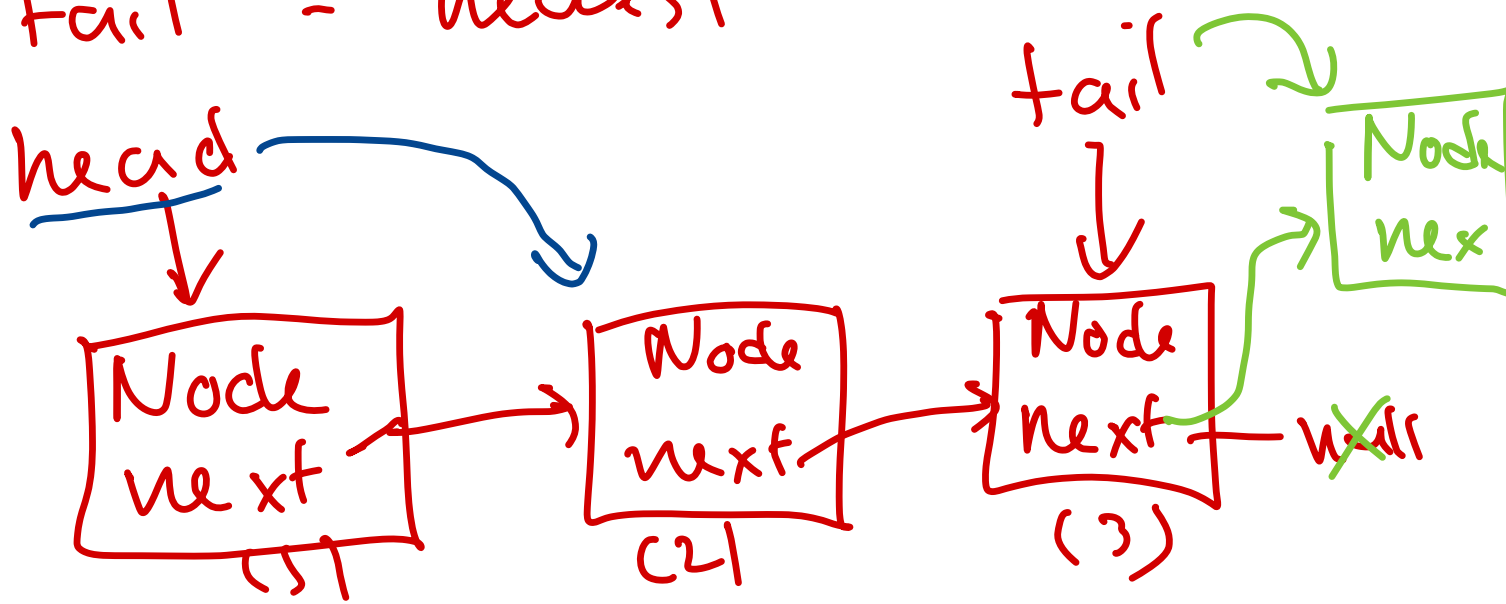
# A Queue Again

**Question.** How to implement a (non-concurrent) queue with a linked list?

enq  $\leftarrow$  add an item  
deq  $\leftarrow$  remove "oldest" item.

(doubly) linked list

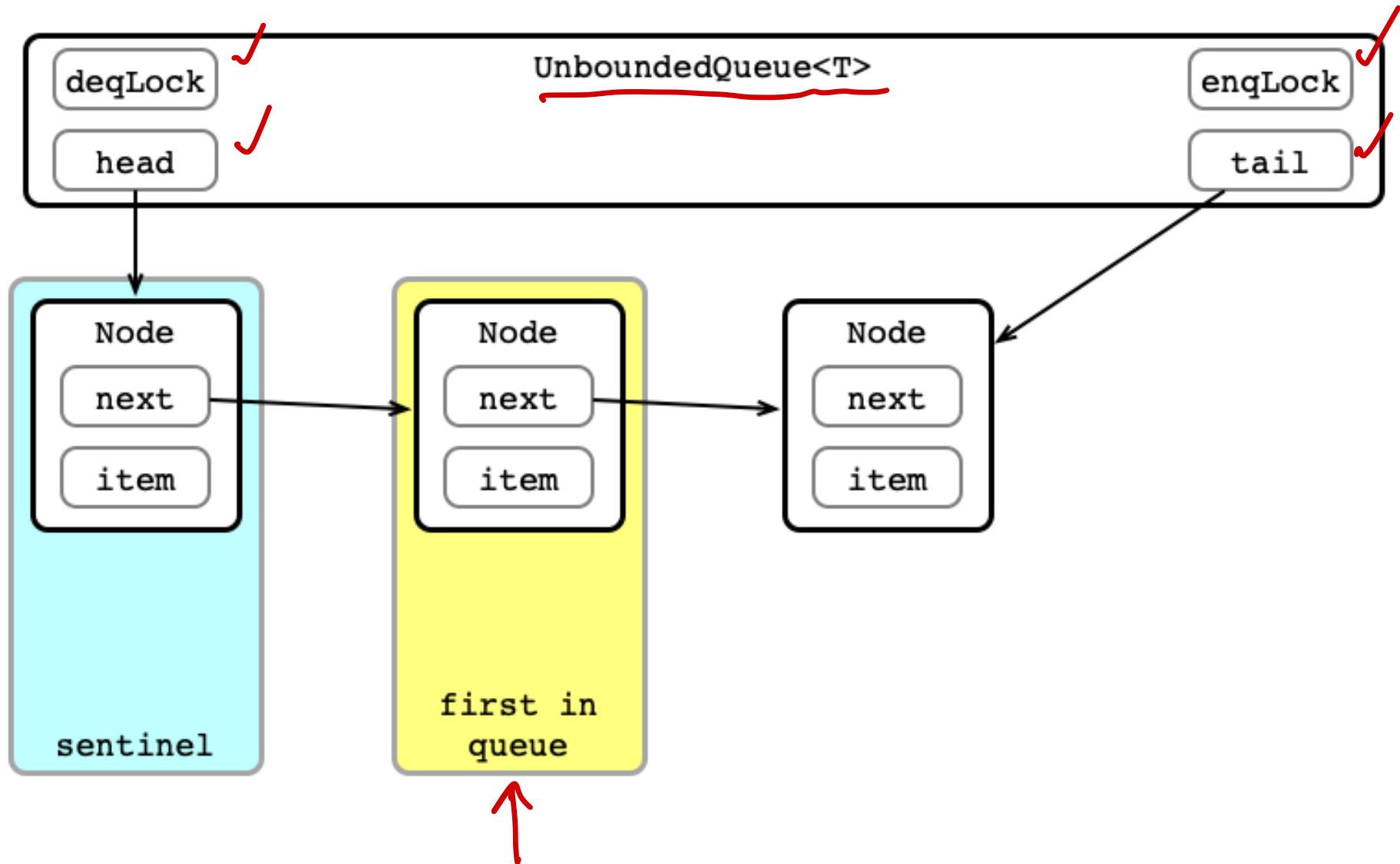
- head = oldest item
- tail = newest



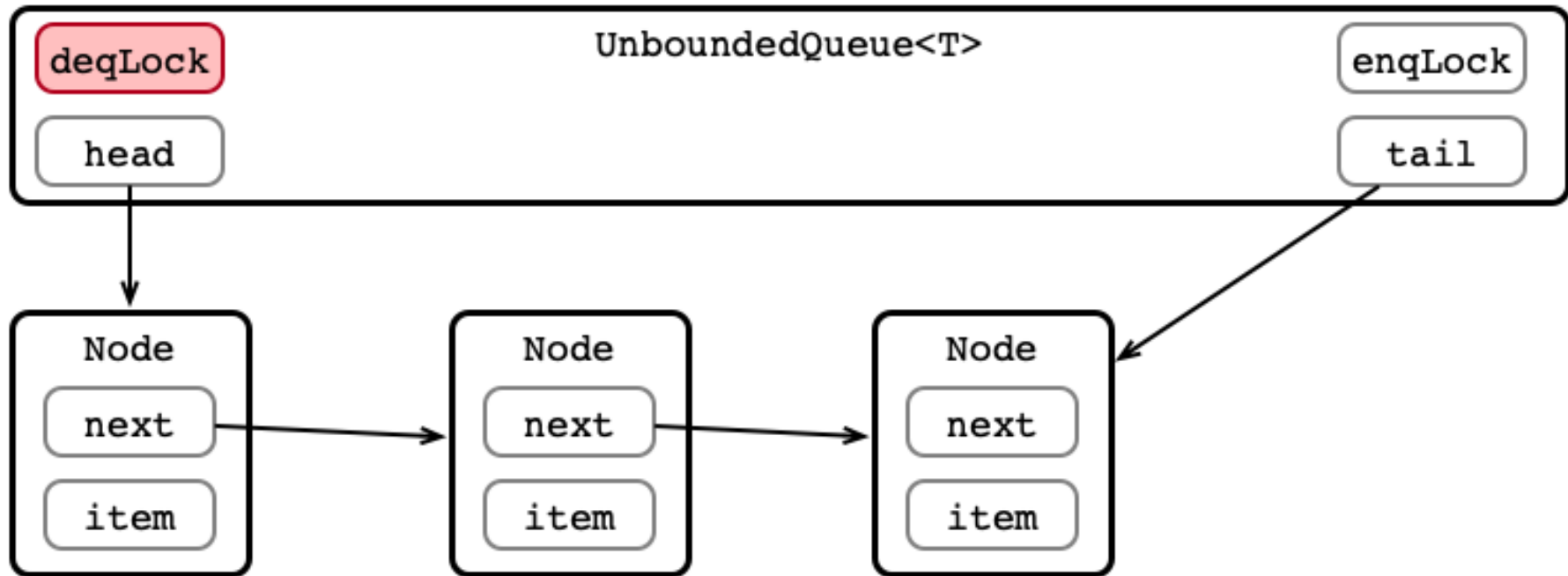
# A Concurrent Queue

- Use linked list implementation of queue
- Store:
  - Node head sentinel
    - `deq` returns `head.next` value (if any), updates head
  - Node tail
    - `enq` updates `tail.next`, updates tail
- Locks:
  - `enqLock` locks `enq` operation
  - `deqLock` locks `deq` operation
  - individual Nodes are *not* locked

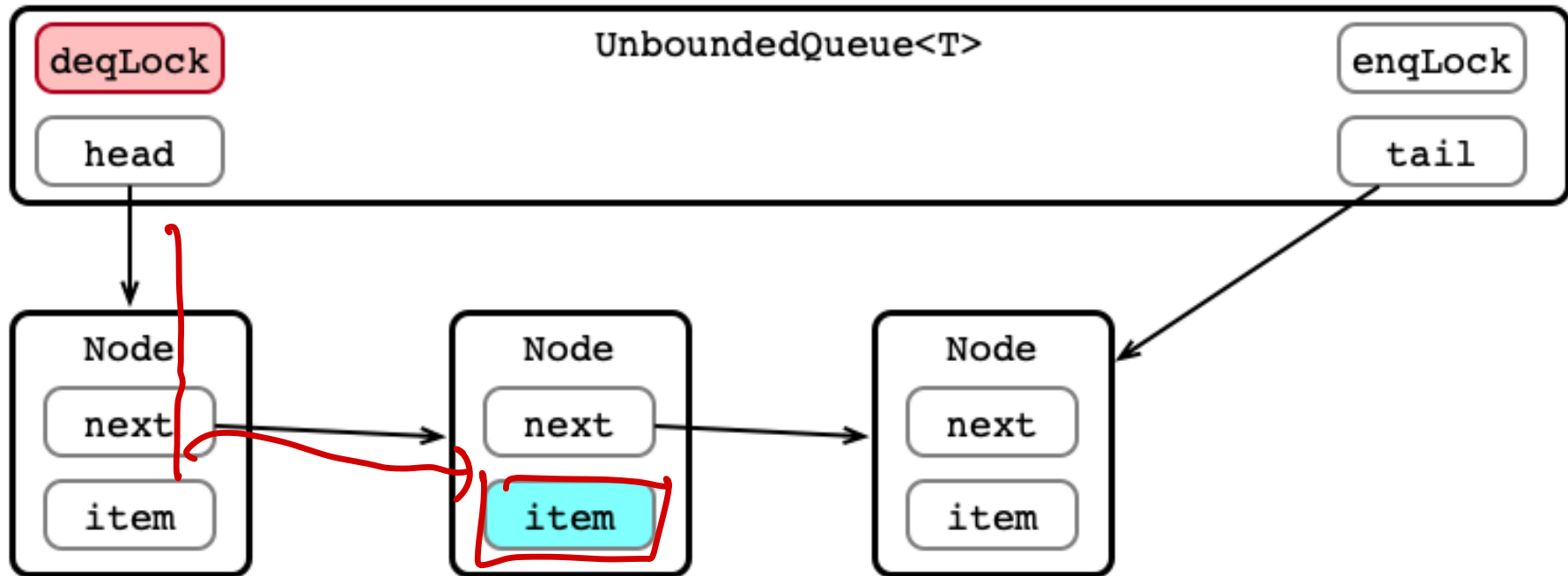
# Unbounded Queue in Pictures



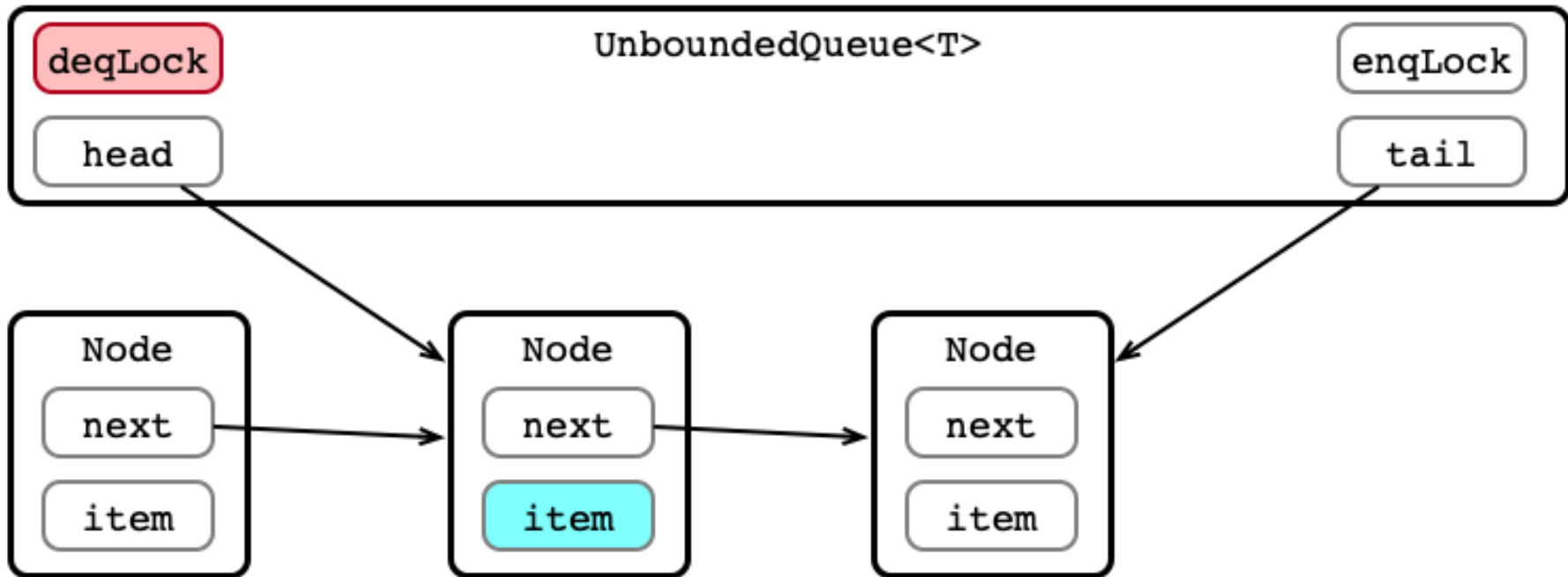
# Dequeue 1: Acquire deqLock



# Deque 2: Get Element (or Exception)

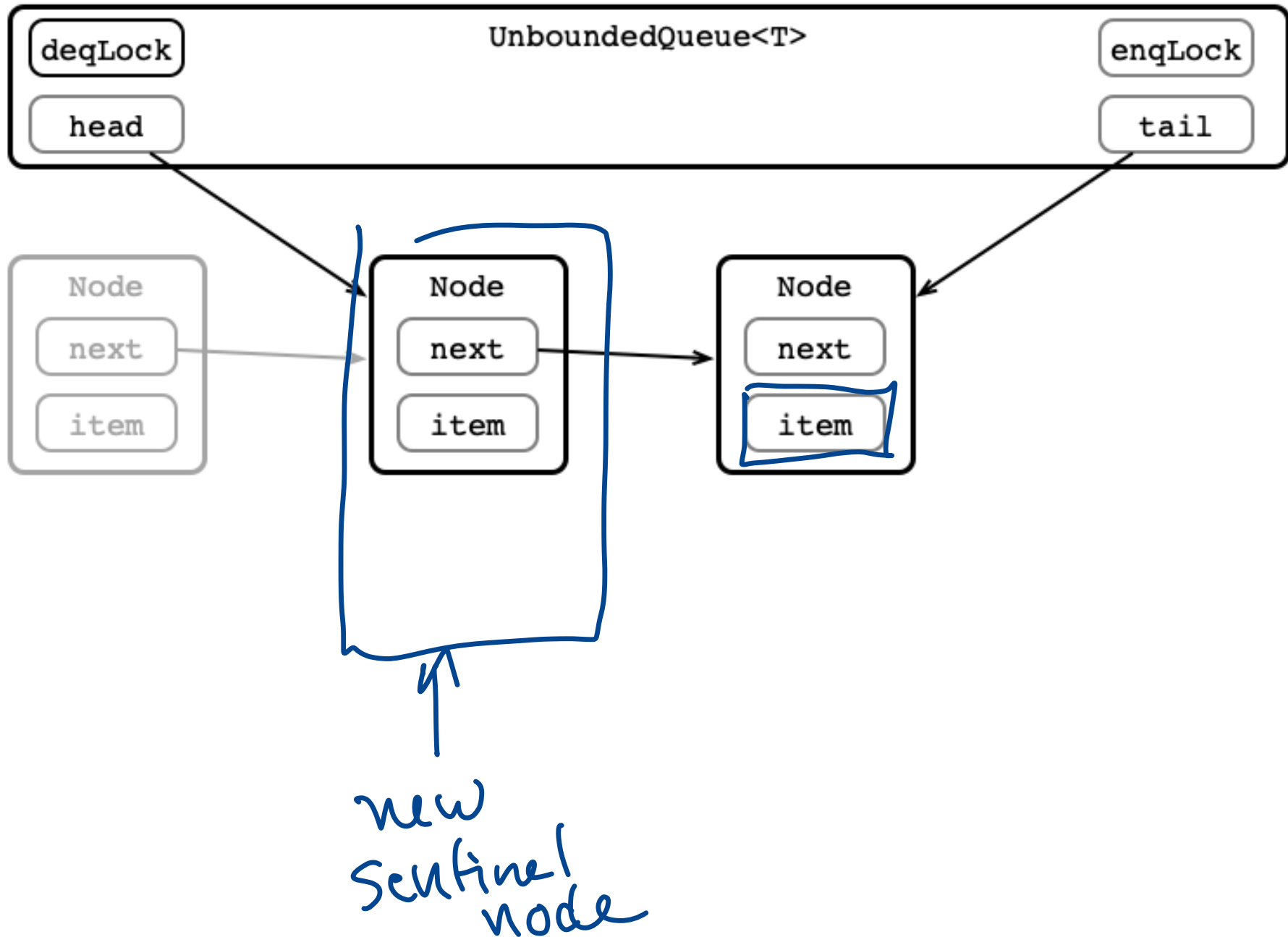


# Dequeue 3: Update head

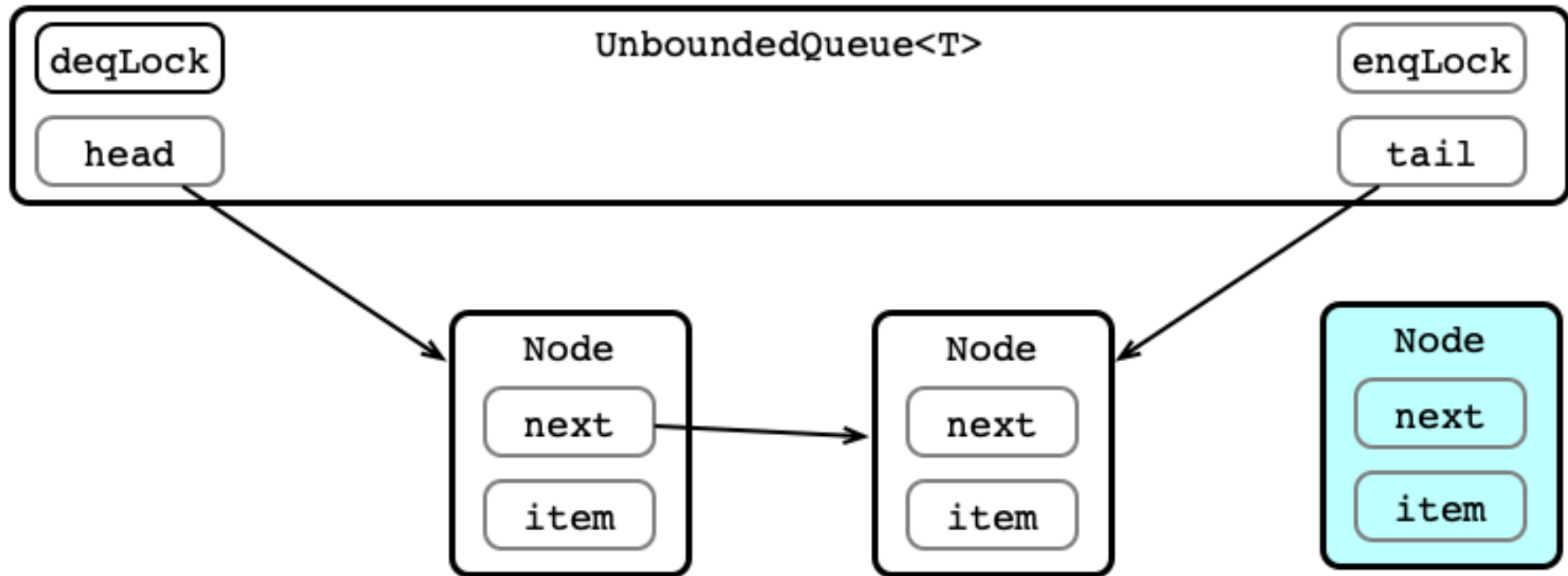




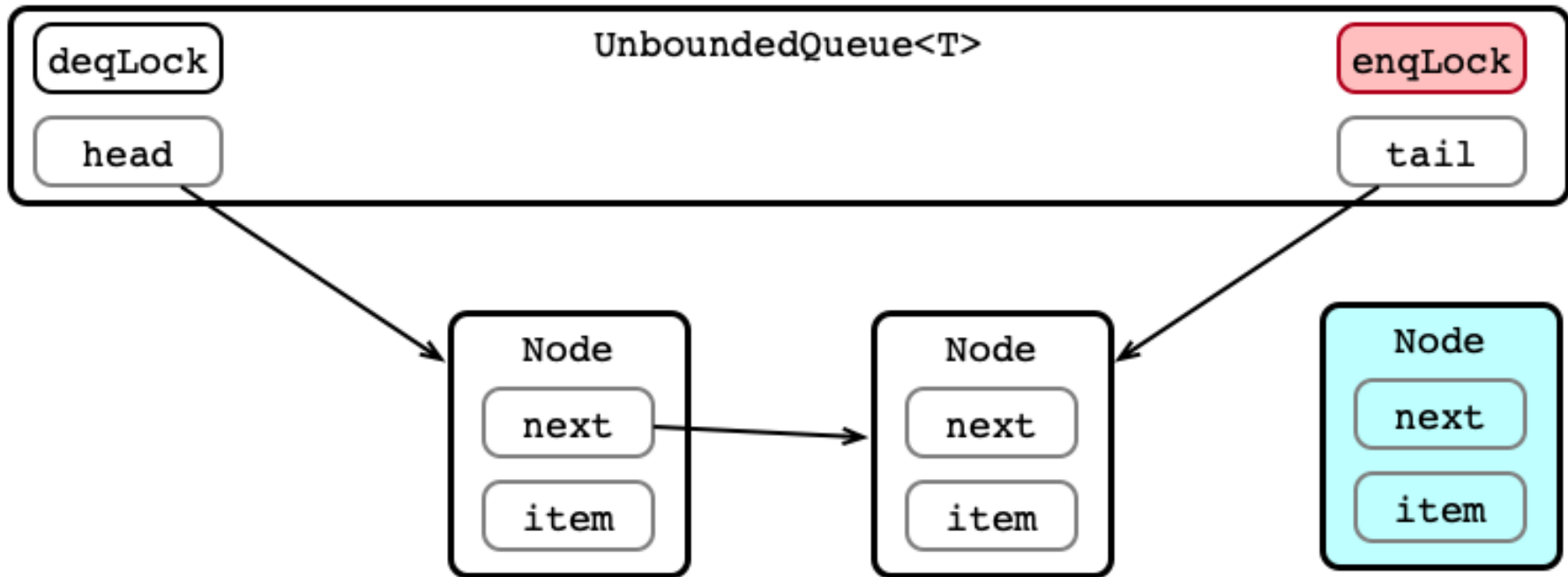
# Dequeue 4: Release Lock



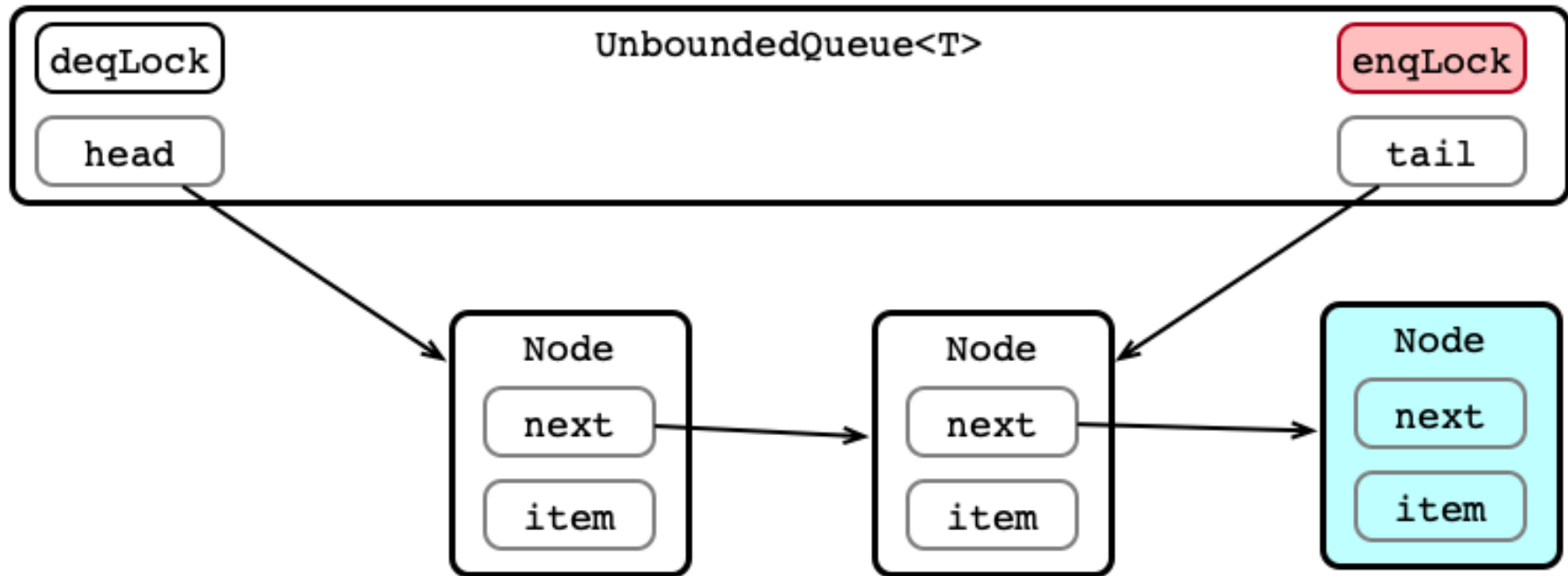
# Enqueue 1: Make Node



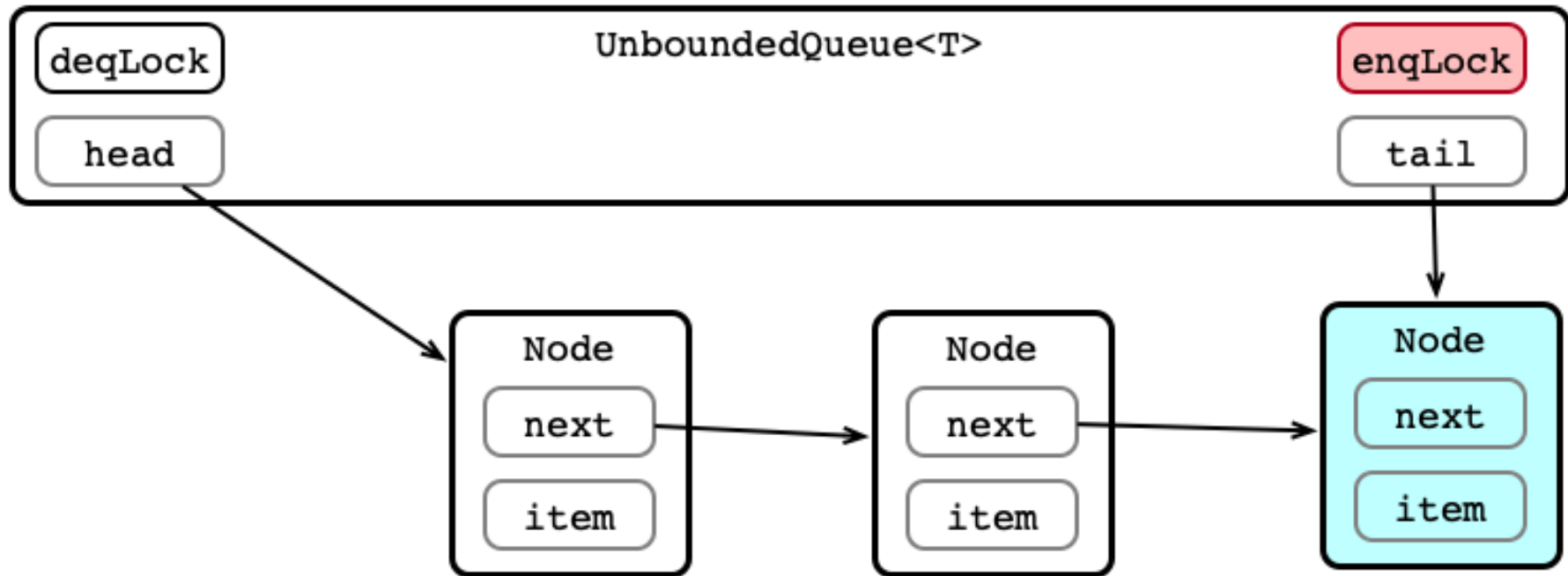
# Enqueue 2: Acquire enqLock



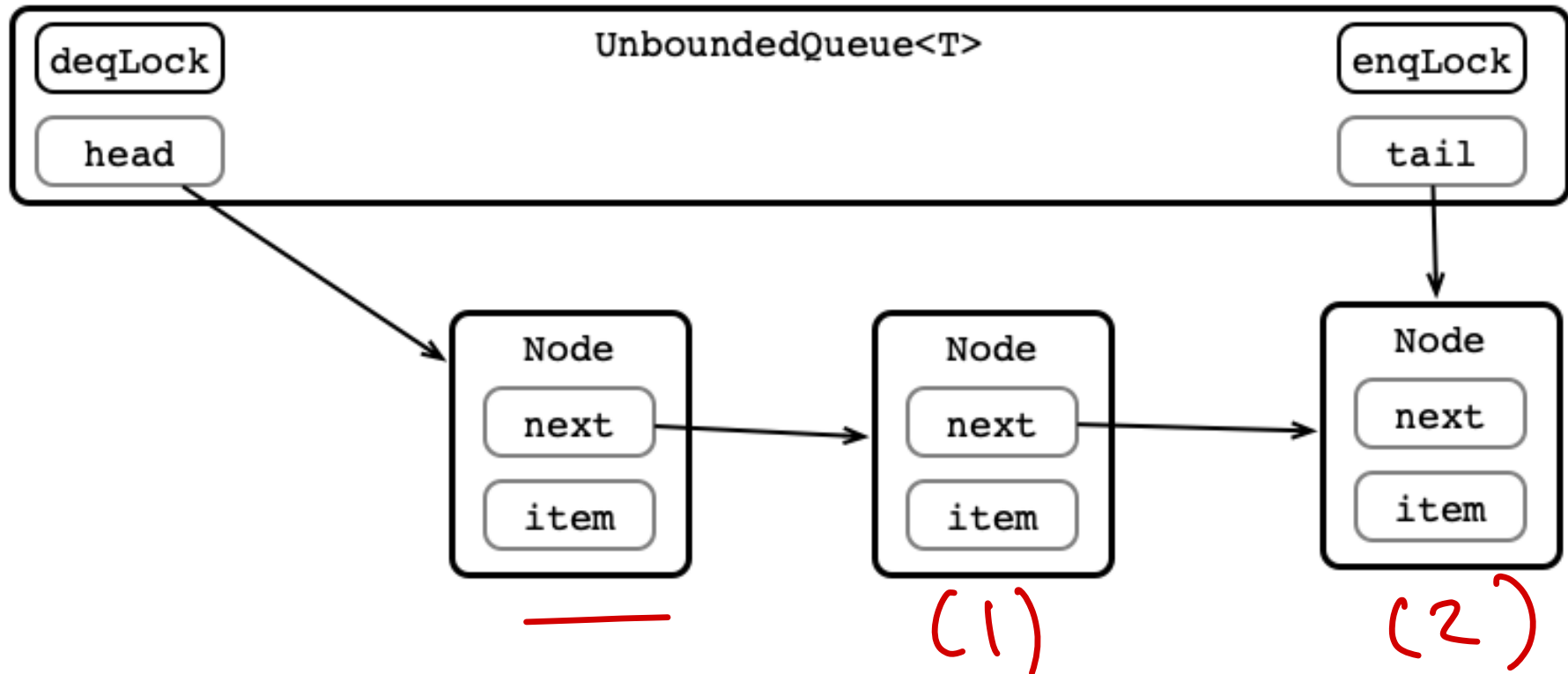
# Enqueue 3: Update tail.next



# Enqueue 4: Update tail



# Enqueue 5: Release Lock



# Question

Why do we need the sentinel node?

# UnboundedQueue in Code

```
public class UnboundedQueue<T> implements SimpleQueue<T> {  
    final ReentrantLock enqLock;  
    final ReentrantLock deqLock;  
    volatile Node head;  
    volatile Node tail;  
  
    public UnboundedQueue() {  
        head = new Node(null); tail = head;  
        enqLock = new ReentrantLock();  
        deqLock = new ReentrantLock(); }  
  
    ...  
}
```



# Node Class

```
class Node {  
    final T value;  
    volatile Node next;  
  
    public Node (T value) {  
        this.value = value;  
    }  
}
```

# enq Method

```
public void enq (T value) {
    enqLock.lock();
    try {
        Node nd = new Node(value);
        tail.next = nd;
        tail = nd;
    } finally {
        enqLock.unlock();
    }
}
```

# deq Method

```
public T deq() throws EmptyException {
    T value;
    deqLock.lock();
    try {
        if (head.next == null){throw new EmptyException();}
        value = head.next.value;
        head = head.next;
        return value;
    } finally {
        deqLock.unlock();
    }
}
```

# Is UnboundedQueue Linearizable?

1. What concurrent operations do we need to consider?
2. What internal states do we need to consider?
3. What are the linearization points (if any)?

# Pertinent Lines

```
public void enq (T value) {
    Node nd = new Node(value);
    tail.next = nd;
    tail = nd;
}

public T deq() throws EmptyException {
    if (head.next == null){throw new EmptyException();}
    value = head.next.value;
    head = head.next;
    return value;
}
```

# Next Time

Concurrent queues without locks?!?!