Lecture 16: Drawing General Graphs II

COSC 225: Algorithms and Visualization

Spring, 2023

Announcements

1. Assignment 07 posted, due Friday
2. Quiz Next Monday ←
    - Apply Tidy Tree algorithm by hand
3. Assignment 08 posted soon, due next Friday
4. Limited OH This Week (advising week)
    - No OH today
    - No OH on Thursday

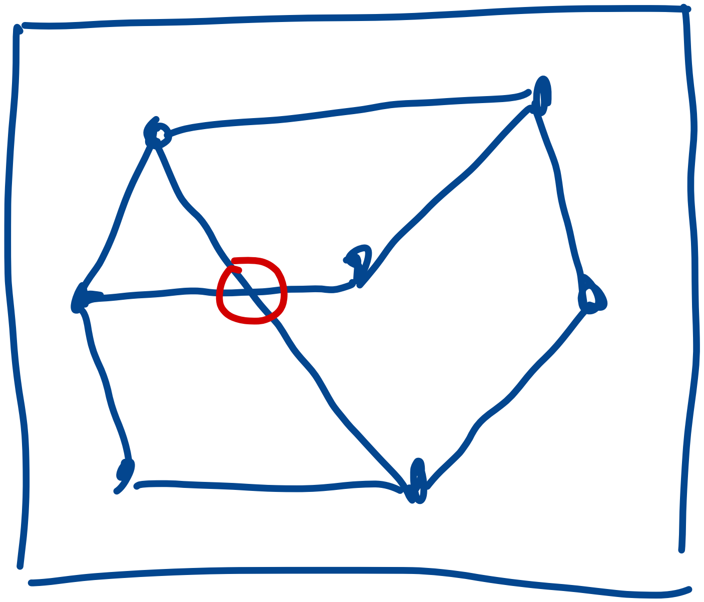
Outline

1. AVSDF Circular Layouts
2. Force-directed Graph Layout

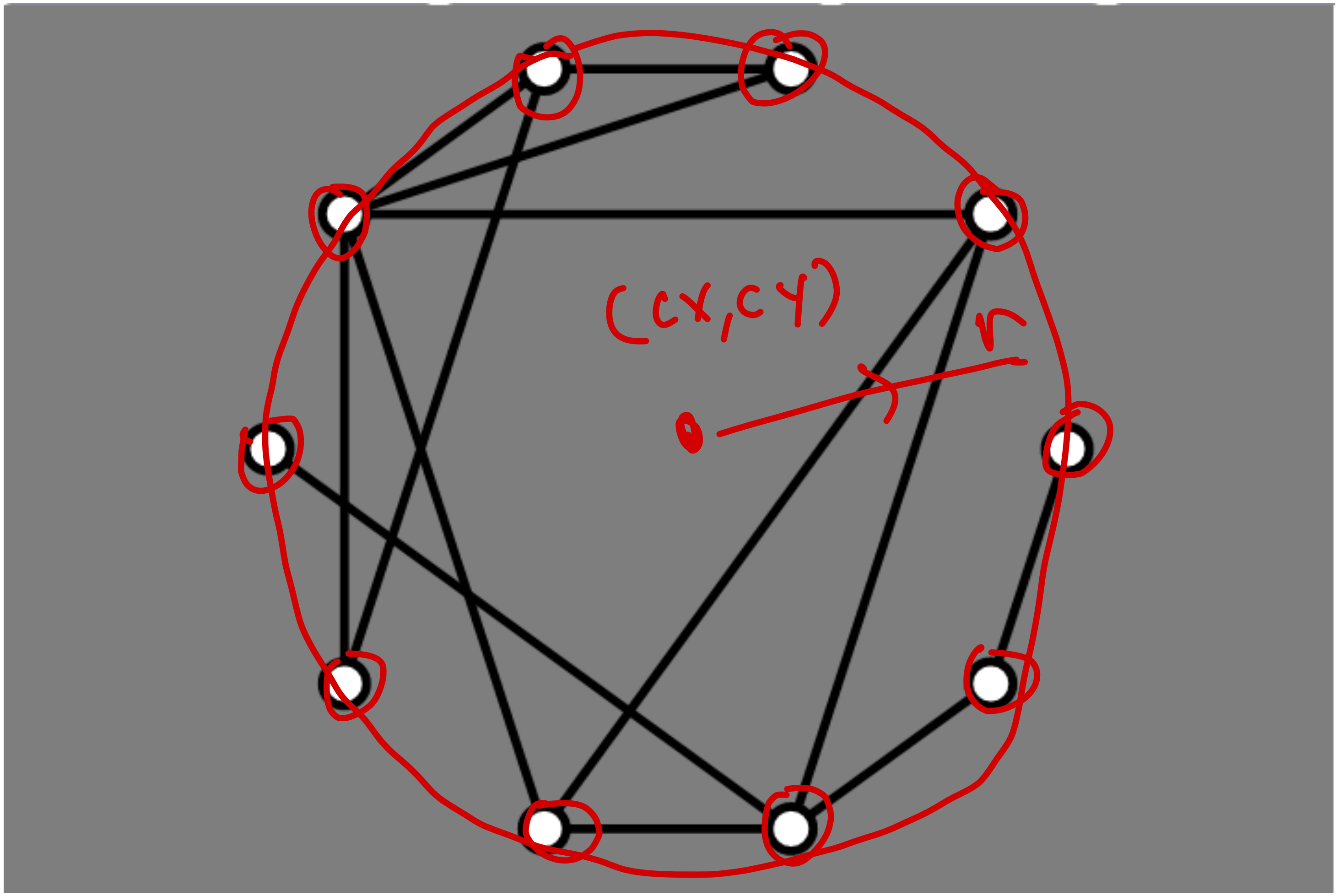
Goals

From Fruchterman and Reingold (1991):

1. Distribute the vertices evenly in the frame.
2. Minimize edge crossings.
3. Make edge lengths uniform.
4. Reflect inherent symmetry.
5. Conform to the frame.



Last Time: Circular Layouts



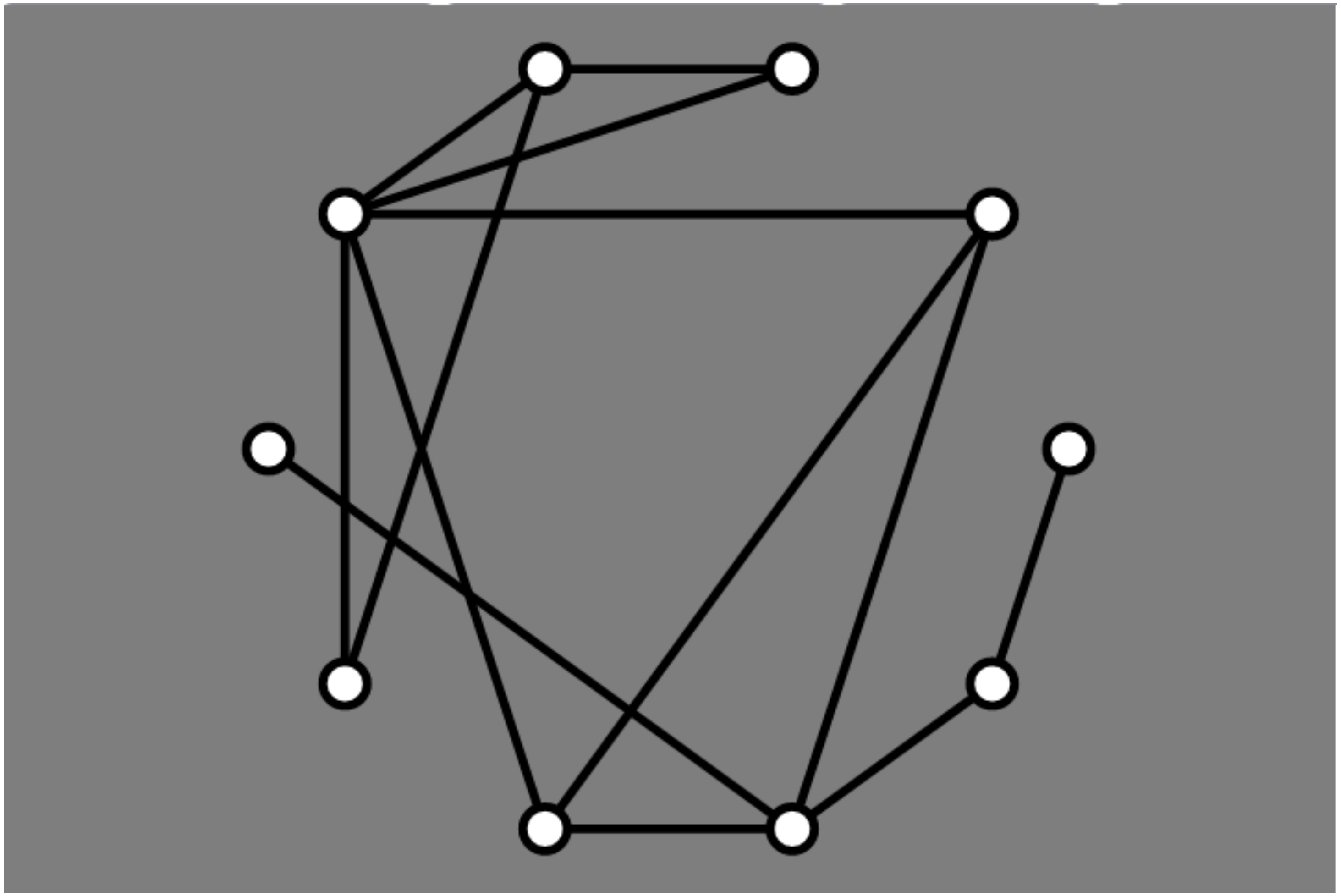
Simpler Challenge: pick an order for vertices

Mäkinen Procedure

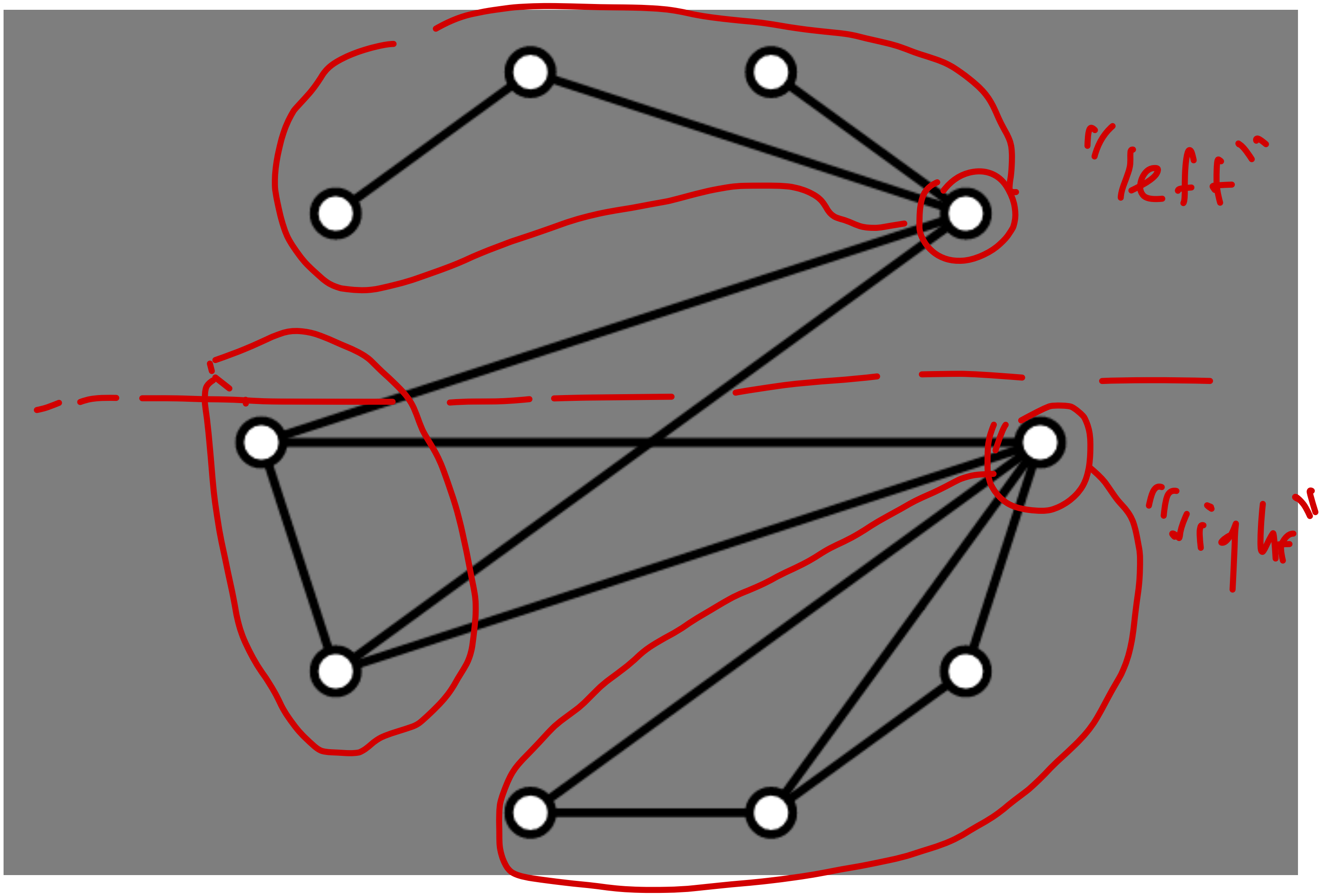
of neighbors

1. Find two vertices of highest degree and add them to `left`/`right` sets
2. Repeat until all vertices are added to `left` or `right`:
    - compute (`right` neighbors) - (`left` neighbors) for each vertex
    - add vertex with largest value to `right`
    - add vertex with smallest value to `left`
3. Add `left` vertices on left side, `right` on right side

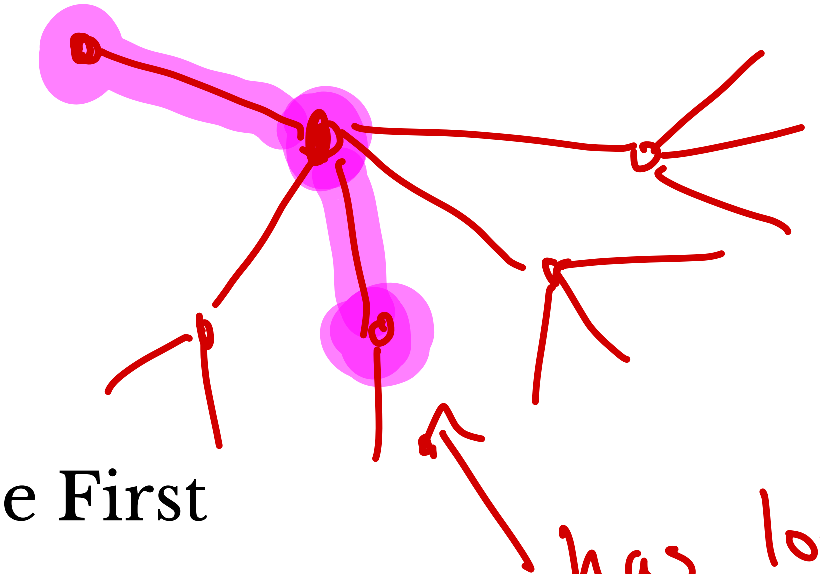
Mäkinen Results: Before



Mäkinen Results: After



AVSDF Heuristic



has lowest deg of neighbors of first vtx

Adjacent **V**ertex **S**mallest **D**egree **F**irst

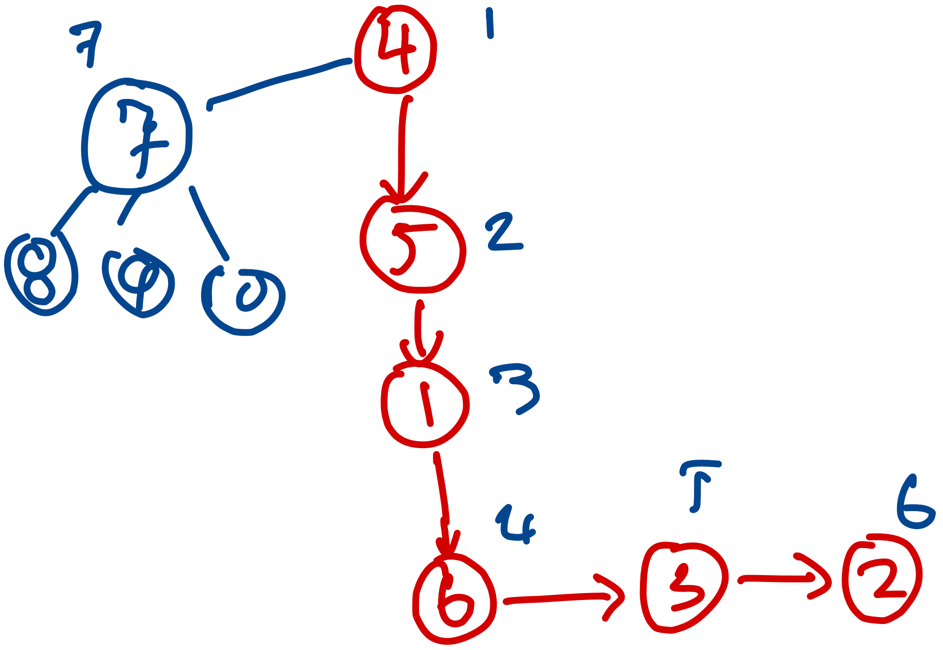
- He & Sykora

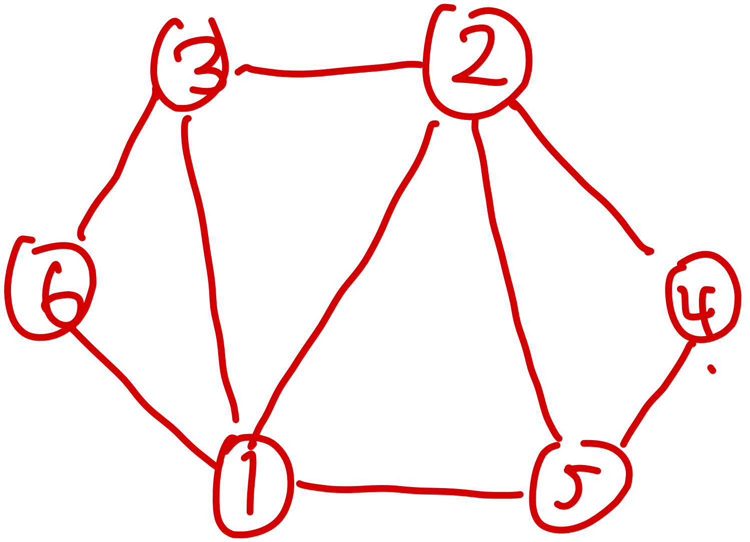
Idea:

- perform depth-first search, starting from vertex of minimal degree
- always explore minimum degree neighbor first

AVSDF Example

```
1: 2, 6, 3, 5
2: 1, 3, 5, 6
3: 1, 2, 6
4: 2, 5
5: 1, 2, 4
6: 1, 3
```





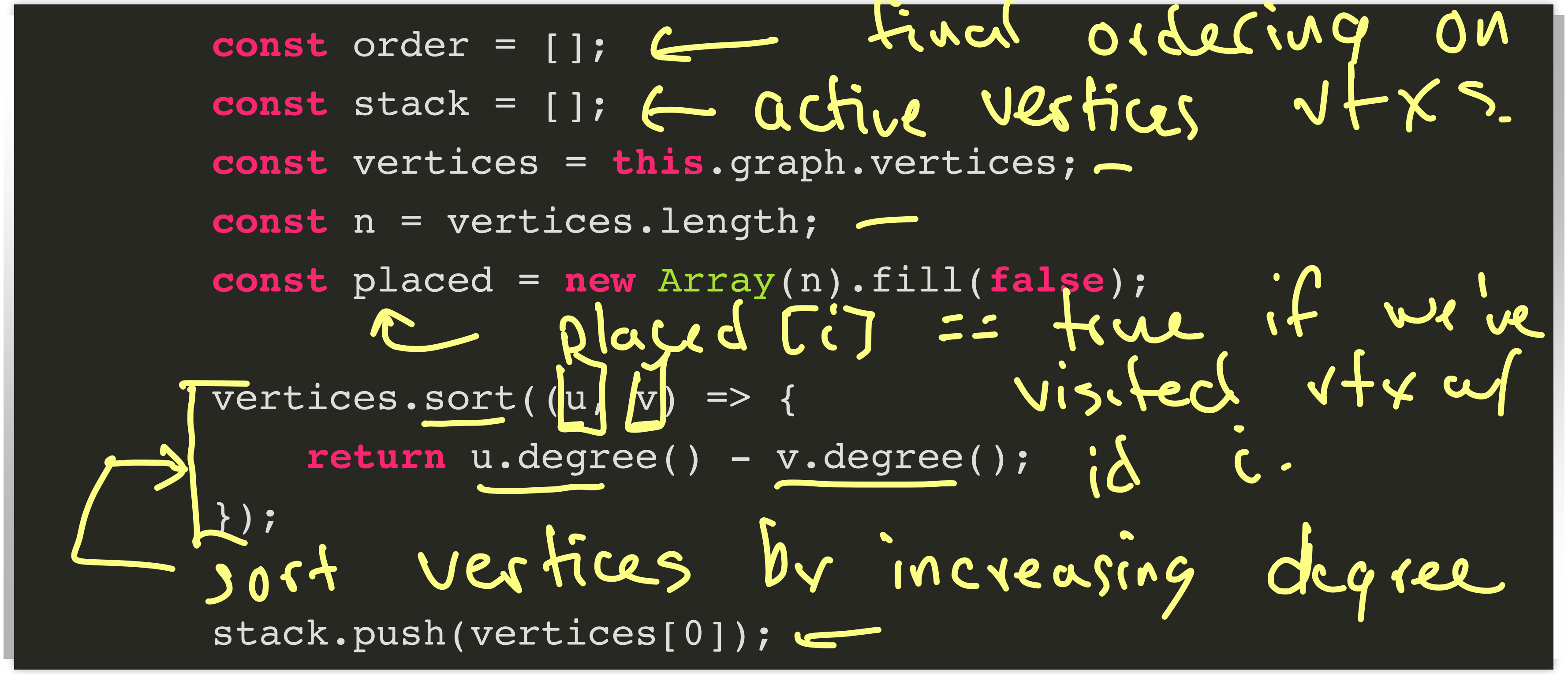
How To Implement AVSDF Efficiently

- What do we keep track of and store?
- How do we update data structures?
- How efficient is the procedure

→ for DFS maintain a stack

→ array / list / whatever of visited nodes in order visited

AVSDF Initialization



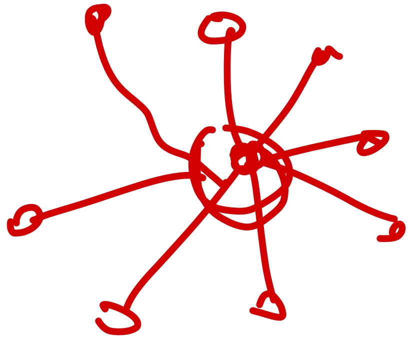
```
const order = [];
const stack = [];
const vertices = this.graph.vertices;
const n = vertices.length;
const placed = new Array(n).fill(false);

vertices.sort((u, v) => {
    return u.degree() - v.degree();
});

stack.push(vertices[0]);
```

final ordering on

active vertices vtxs.

placed[i] == true if we've visited vtx w/ id i.

sort vertices by increasing degree



Main Loop

```
while (stack.length > 0) {
  let vtx = stack.pop();
  if (!placed[vtx.id]) {
    order.push(vtx);
    placed[vtx.id] = true;
    vtx.neighbors.sort((u, v) => {
      return v.degree() - u.degree();
    });
    for (let nbr of vtx.neighbors) {
      if (!placed[nbr.id]) { stack.push(nbr); }
}}}
```

current vtx

have not yet placed vtx

k log k

k nbrs

sort vtx's nbrs largest to smallest deg

push nbrs onto stk largest to smallest deg.

each vtx only popped off stack once

n^2 ?

$\leq$

$\boxed{n^2 \log n}$ ←

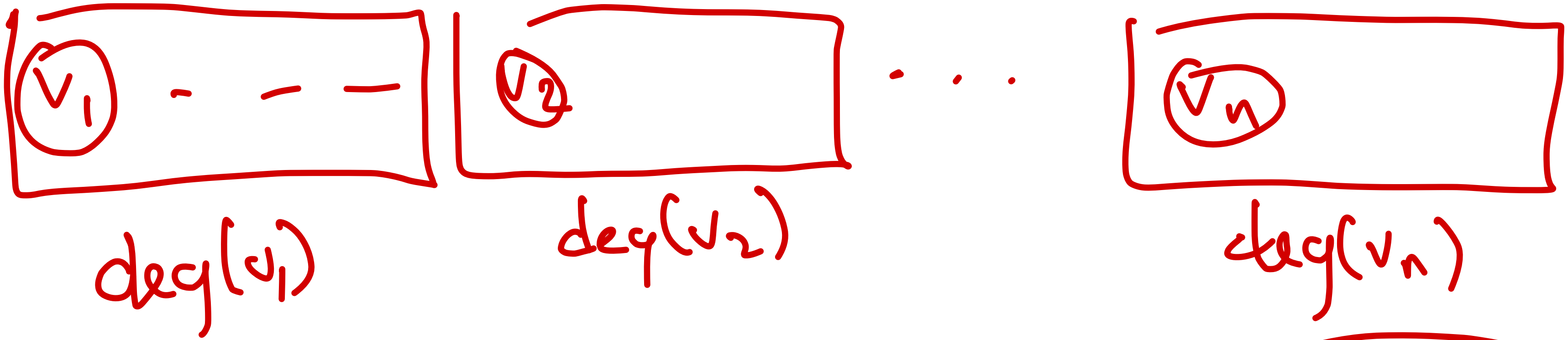
$\boxed{m \log m^n}$

$m \leq n^2$

$\log m \leq \log n^2$

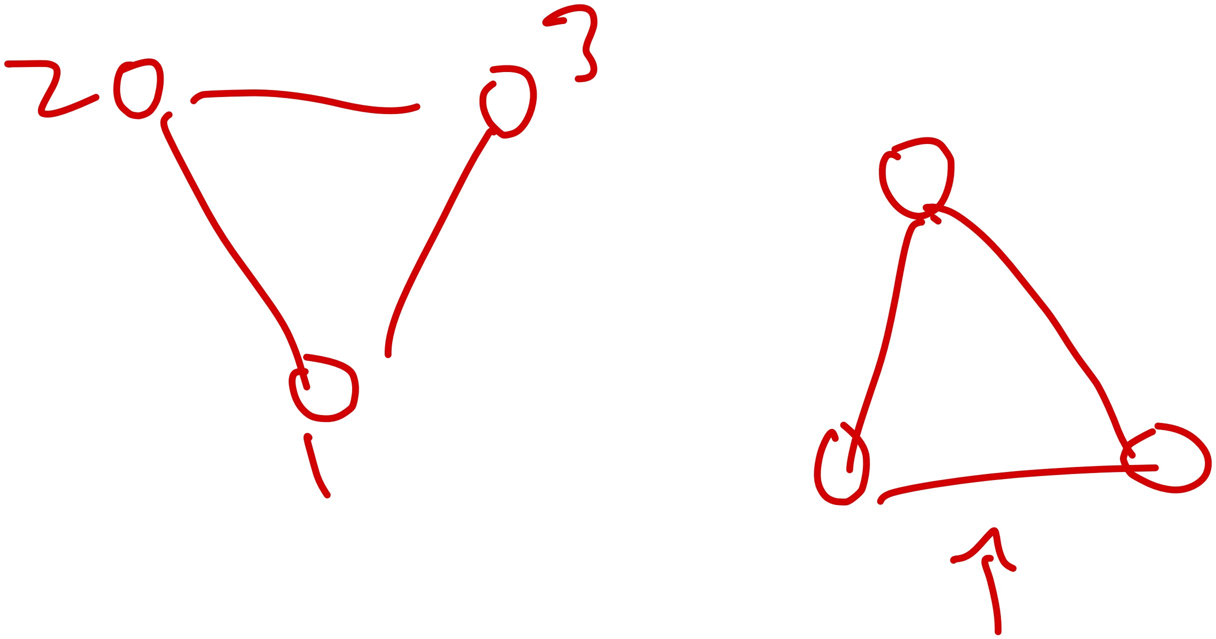
Running Time of Main Loop?

n vertices, m edges



$$\boxed{deg(v_1)} + \boxed{deg(v_2)} + \cdots + \boxed{deg(v_n)} = 2m$$

When Will Algorithm Fail?

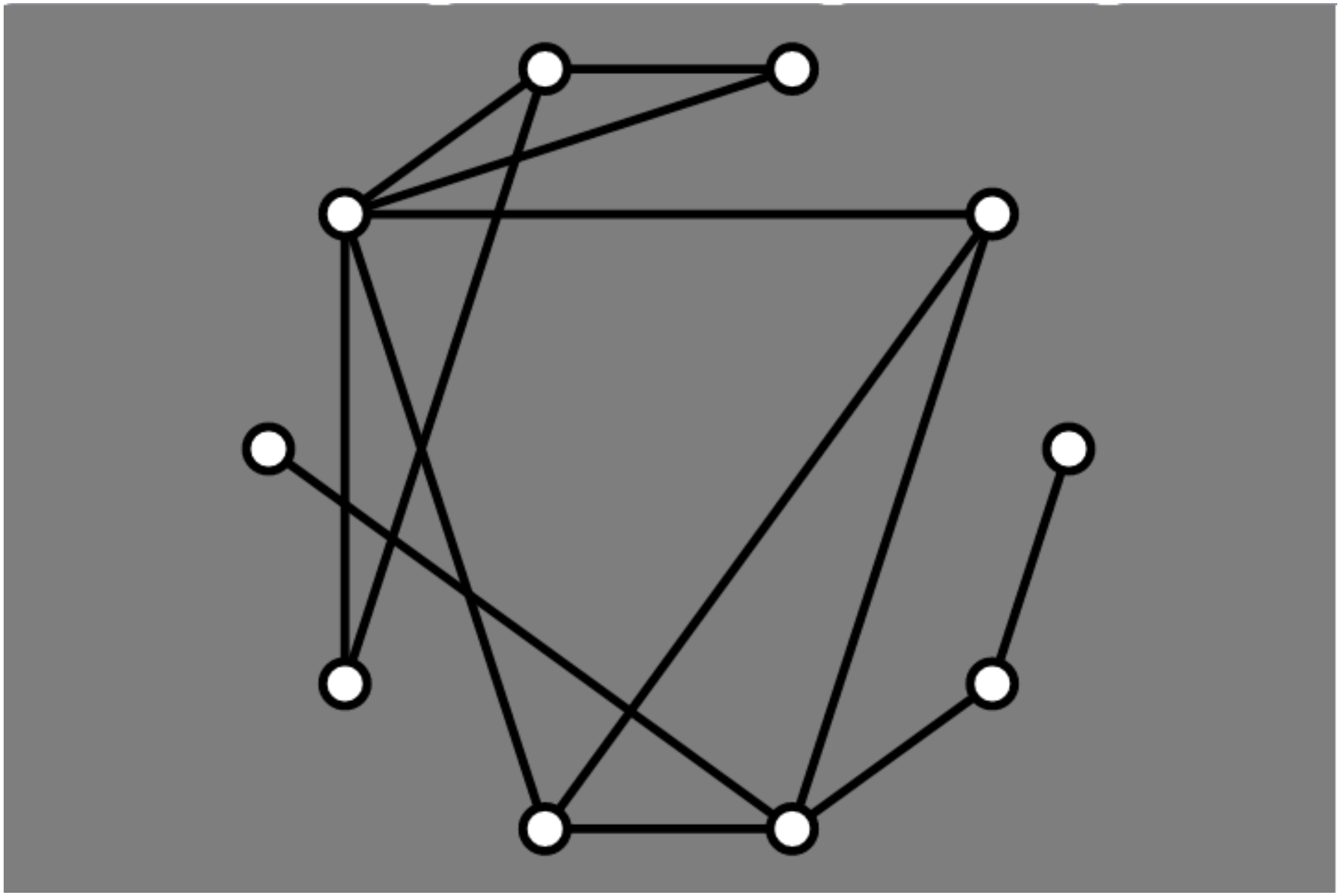


won't get drawn

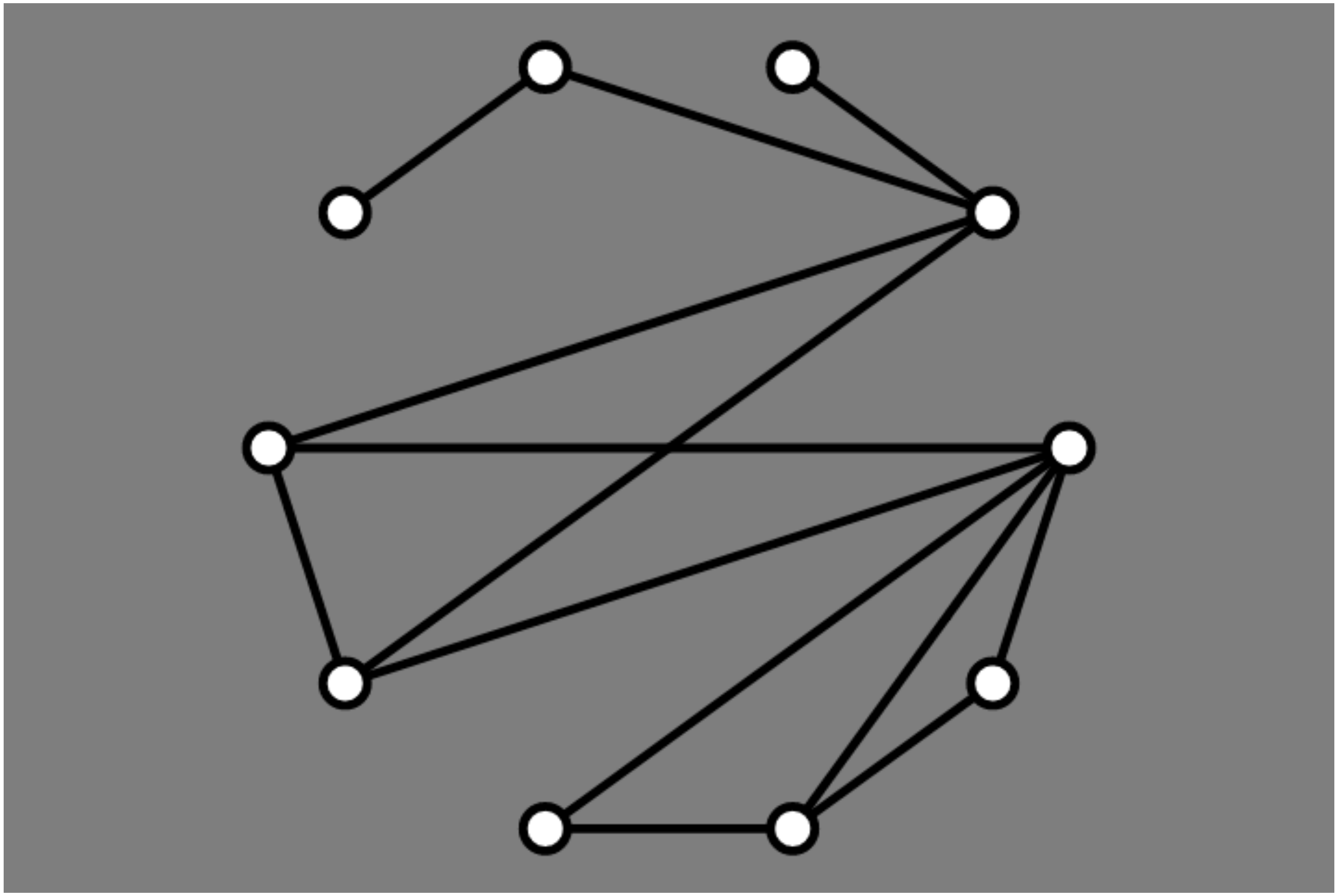
If graph is disconnected

fix → run same procedure for each connected comp.

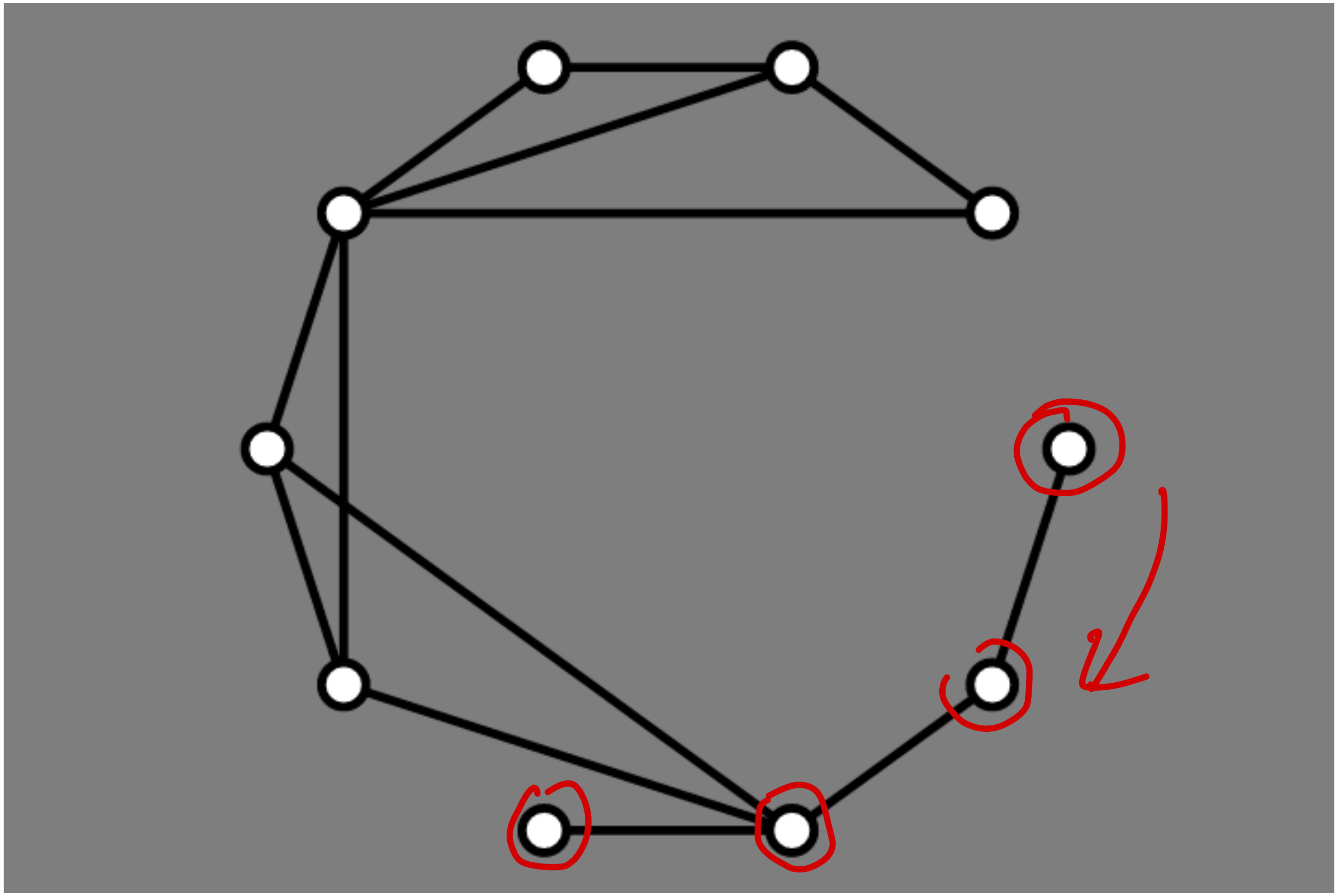
1: Random Circular



2: Mäkinen Circular



3: AVSDF Circular



AVSDF Demo

Force-Directed Layout

A Different Approach

Don't place vertices *explicitly*

Instead:

- associate graph with a physical system
- simulate the physical system
- let system evolve
- place vertices at final location according to evolution

Goals, Again

From Fruchterman and Reingold (1991):

1. Distribute the vertices evenly in the frame.
2. Minimize edge crossings.
3. Make edge lengths uniform.
4. Reflect inherent symmetry.
5. Conform to the frame.

vertices repel one another

edges pull endpoints

Physical Simulation

1. All vertices should repel each other

2. Adjacent vertices should attract each other

Due to:

- Eades, 1984
- Fruchterman and Reingold, 1991 ←
    - we'll follow this paper

Competing Forces

0 ← dist → 0

k some param

All vertices:

```
function repulsiveForce (dist, k) {
  return k * k / dist;
}
```

farther apart ⇒ less repulsion

```
function attractiveForce (dist, k) {
  return dist * dist / k;
}
```

farther apart ⇒ stronger pull

Question

When do attractive and repulsive forces cancel out for adjacent vertices?

```
function repulsiveForce (dist, k) {
  return k * k / dist;
}

function attractiveForce (dist, k) {
  return dist * dist / k;
}
```

$$\frac{k * k}{dist} = \frac{dist * dist}{k}$$

$$\Leftrightarrow \quad k = dist$$

F&R Main Loop

For each vertex:

- compute net *force* on that vertex
  - find repulsive contribution from each other vertex
  - find attractive contribution from each neighbor
  - sum all contributions
- move each vertex according to net force
  - move in direction of net force
  - amount is min of net force and “temperature”
- update temperature ←

Repeat until “done”

max movement
under any
force

Setting Parameters

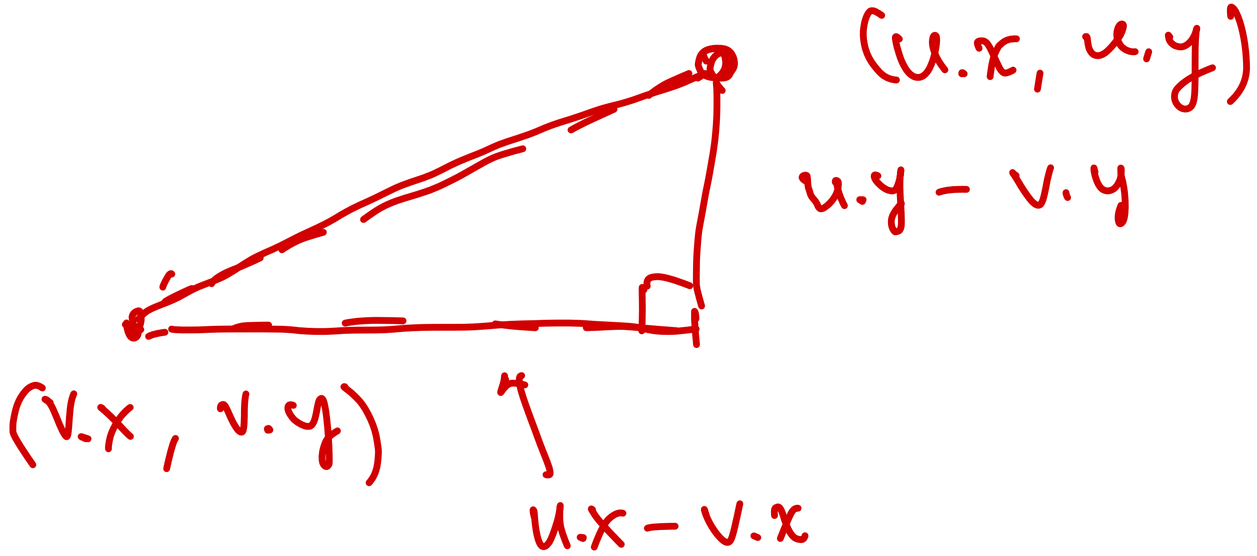
Want `k` is “ideal” distance between vertices

- `area = width * height`
- `n = vertices.length`
- `k = C * Math.sqrt(area / n)`
  - `k` is “typical” distance if vertices are spread out
  - `C` some constant to be determined

Computing Forces I

- v at point `(v.x, v.y)`
- u at point `(u.x, u.y)`

What is *distance* from v to u?



$$dist = \sqrt{(u.x - v.x)^2 + (u.y - v.y)^2}$$

Computing Forces I

- v at point `(v.x, v.y)`
- u at point `(u.x, u.y)`

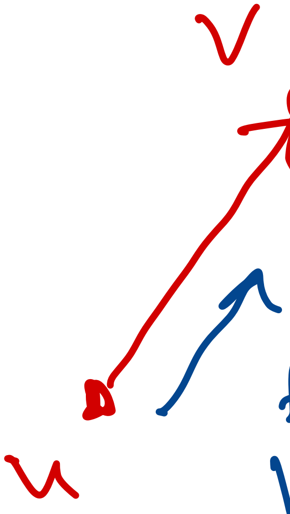
What is *distance* from v to u?

- `deltaX = v.x - u.x`
- `deltaY = v.y - u.y`
- `delta = Math.sqrt(deltaX * deltaX + deltaY * deltaY)`

Computing Forces II

Want (repsulive) force in direction of `(deltaX, deltaY)` with given amount (length): `repulsiveForce(delta, k)`

How to get this?



force has this length

Unit vect. from u to v:

$$\text{repulsive Force}(..) * \left(\frac{\text{deltaX}}{\text{delta}}, \frac{\text{deltaY}}{\text{delta}}\right)$$

Computing Forces II

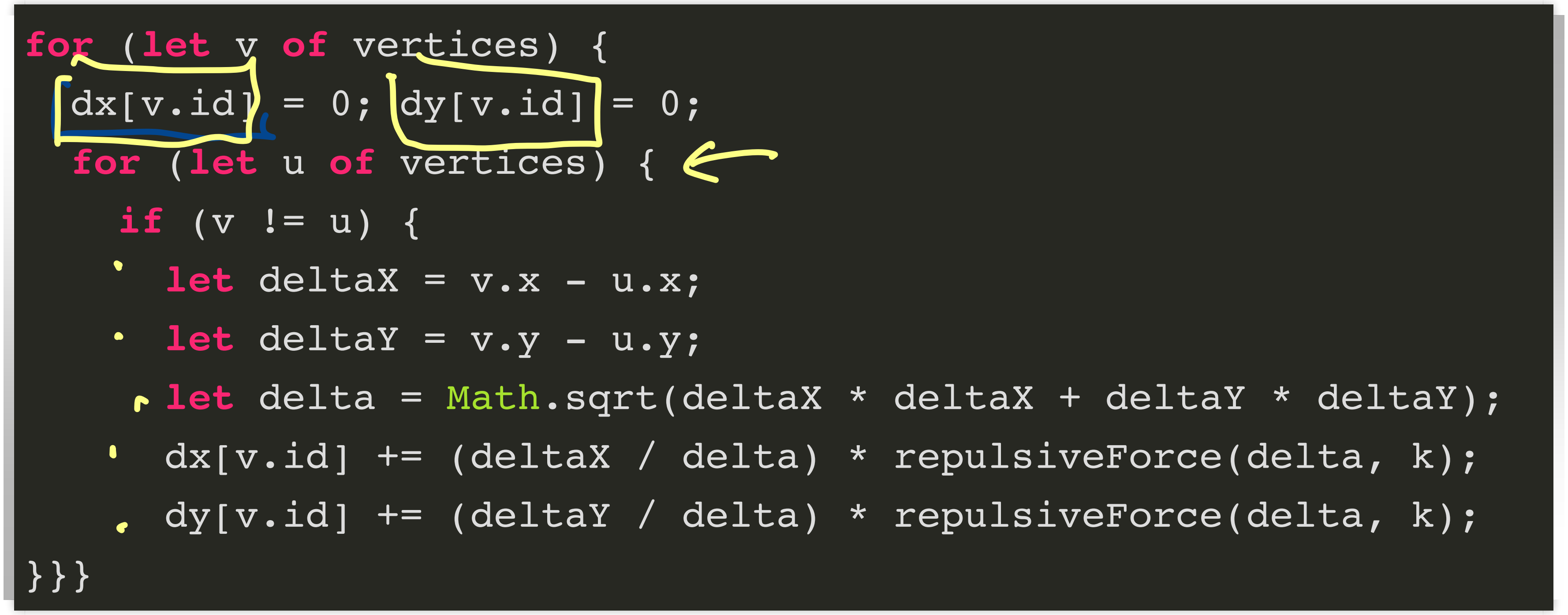
Want (repsulive) force in direction of `(deltaX, deltaY)` with given amount (length): `repulsiveForce(delta, k)`

How to get this?

- `dx = (deltaX / delta) * repulsiveForce(delta, k)`
- `dy = (deltaY / delta) * repulsiveForce(delta, k)`

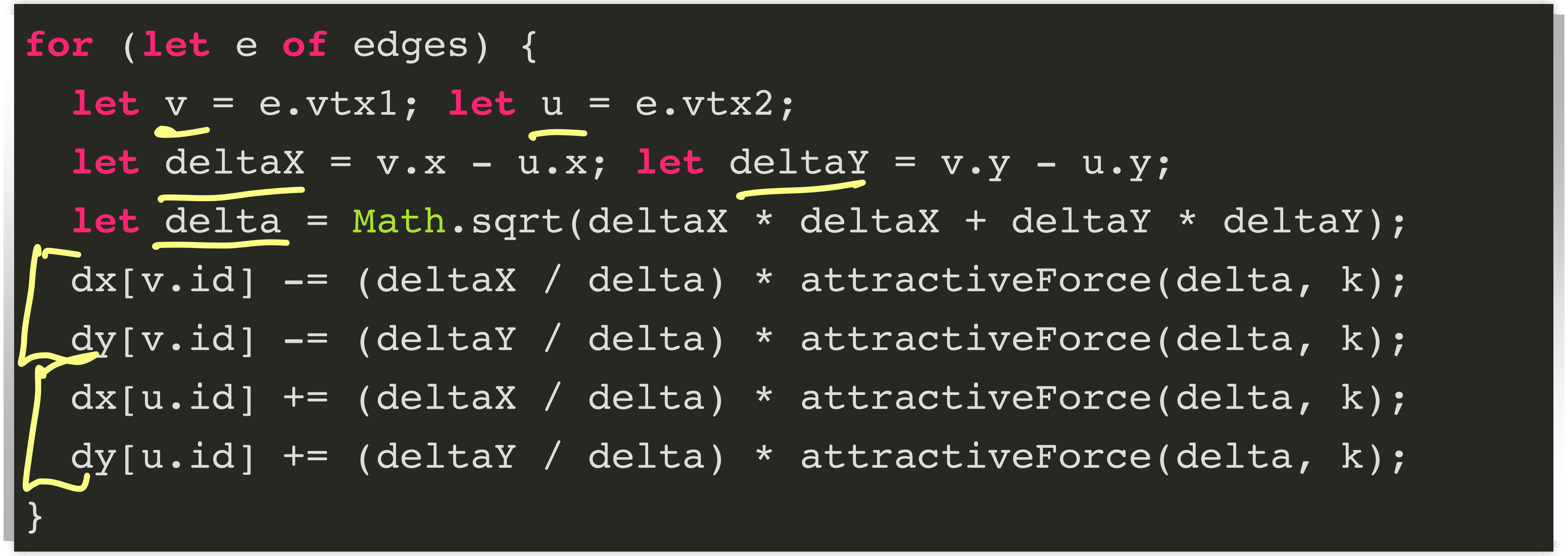
(dx, dy) force applied
to v by u

Adding All Repulsive Contributions



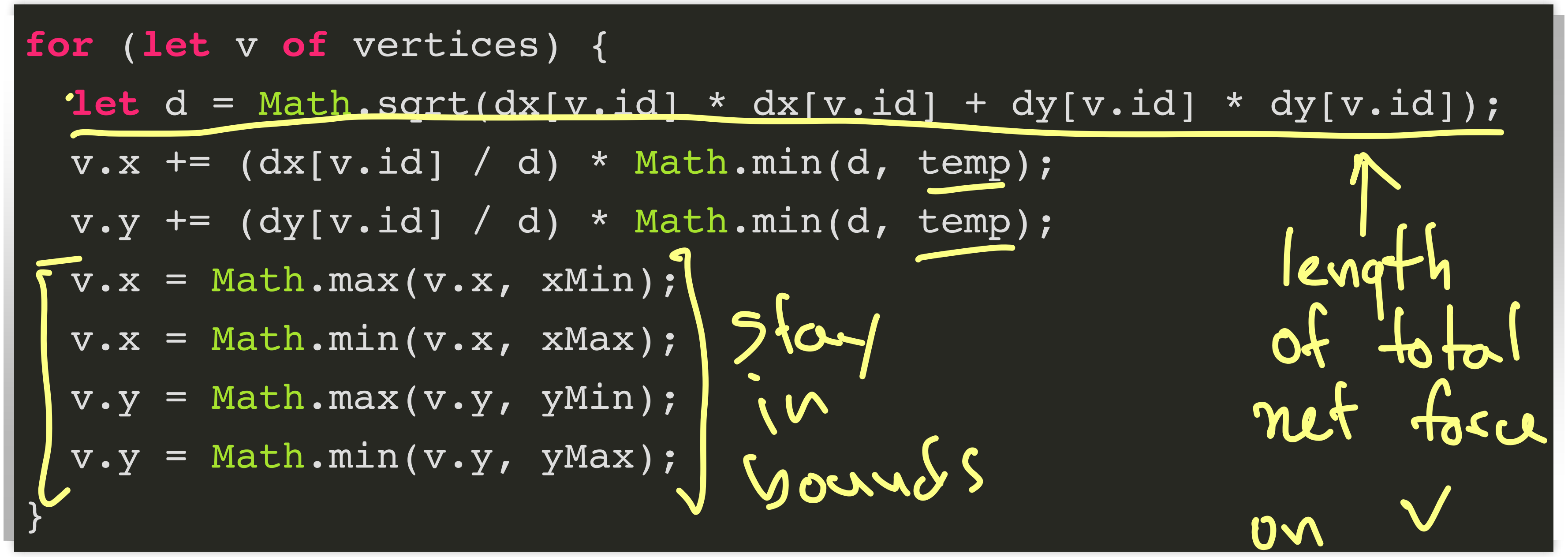
```
for (let v of vertices) {
  dx[v.id] = 0; dy[v.id] = 0;
  for (let u of vertices) {
    if (v != u) {
      let deltaX = v.x - u.x;
      let deltaY = v.y - u.y;
      let delta = Math.sqrt(deltaX * deltaX + deltaY * deltaY);
      dx[v.id] += (deltaX / delta) * repulsiveForce(delta, k);
      dy[v.id] += (deltaY / delta) * repulsiveForce(delta, k);
}}}
```

Similarly For Attractive Forces



```
for (let e of edges) {
  let v = e.vtx1; let u = e.vtx2;
  let deltaX = v.x - u.x; let deltaY = v.y - u.y;
  let delta = Math.sqrt(deltaX * deltaX + deltaY * deltaY);
  dx[v.id] -= (deltaX / delta) * attractiveForce(delta, k);
  dy[v.id] -= (deltaY / delta) * attractiveForce(delta, k);
  dx[u.id] += (deltaX / delta) * attractiveForce(delta, k);
  dy[u.id] += (deltaY / delta) * attractiveForce(delta, k);
}
```

Applying Forces



```
for (let v of vertices) {
  let d = Math.sqrt(dx[v.id] * dx[v.id] + dy[v.id] * dy[v.id]);
  v.x += (dx[v.id] / d) * Math.min(d, temp);
  v.y += (dy[v.id] / d) * Math.min(d, temp);
  v.x = Math.max(v.x, xMin);
  v.x = Math.min(v.x, xMax);
  v.y = Math.max(v.y, yMin);
  v.y = Math.min(v.y, yMax);
}
```

stay in bounds

length of total net force

on ✓

Finally

Repeat:

- update positions
- decrease temperature

Stop when temperature is 0 (or some fixed number of iterations)

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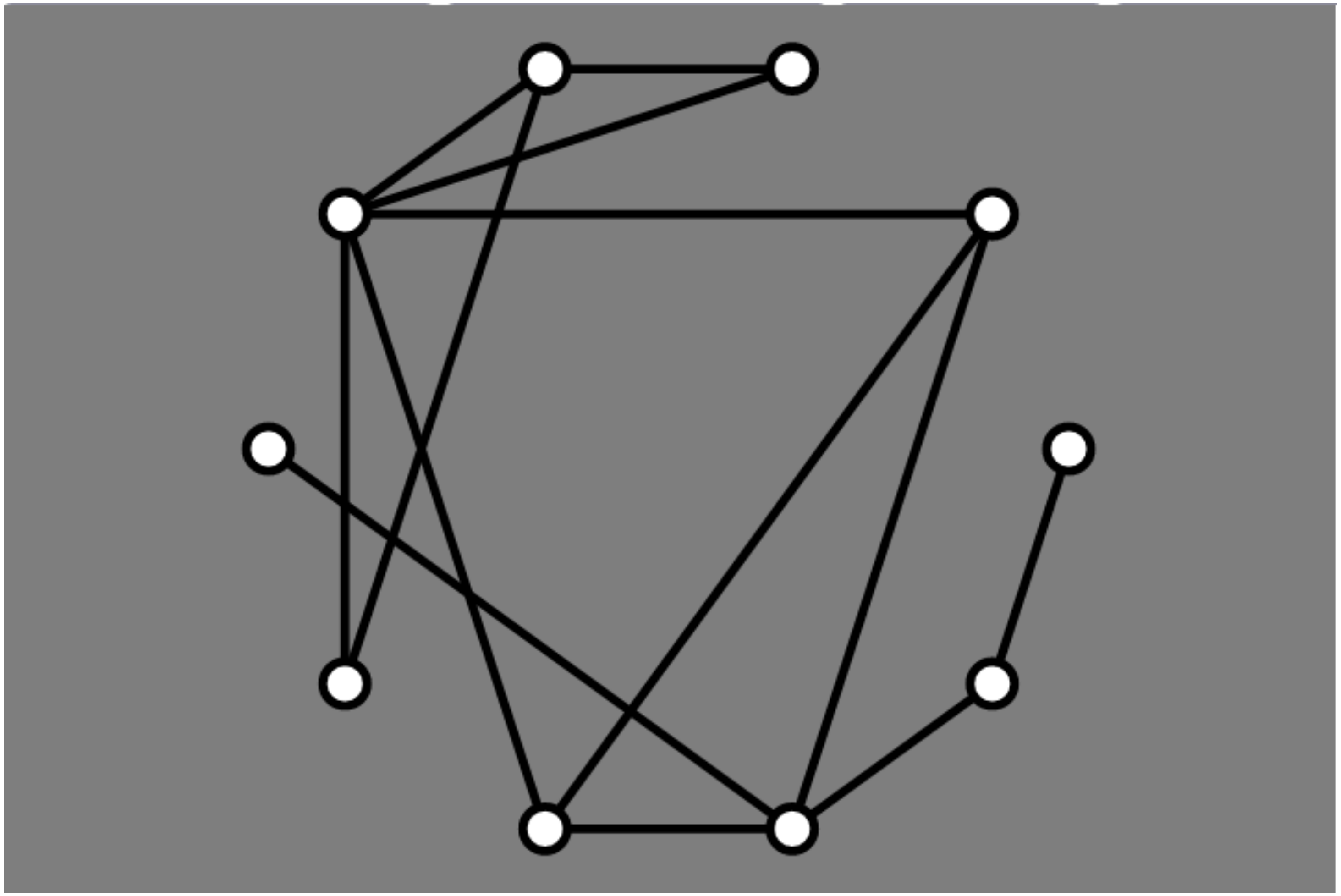

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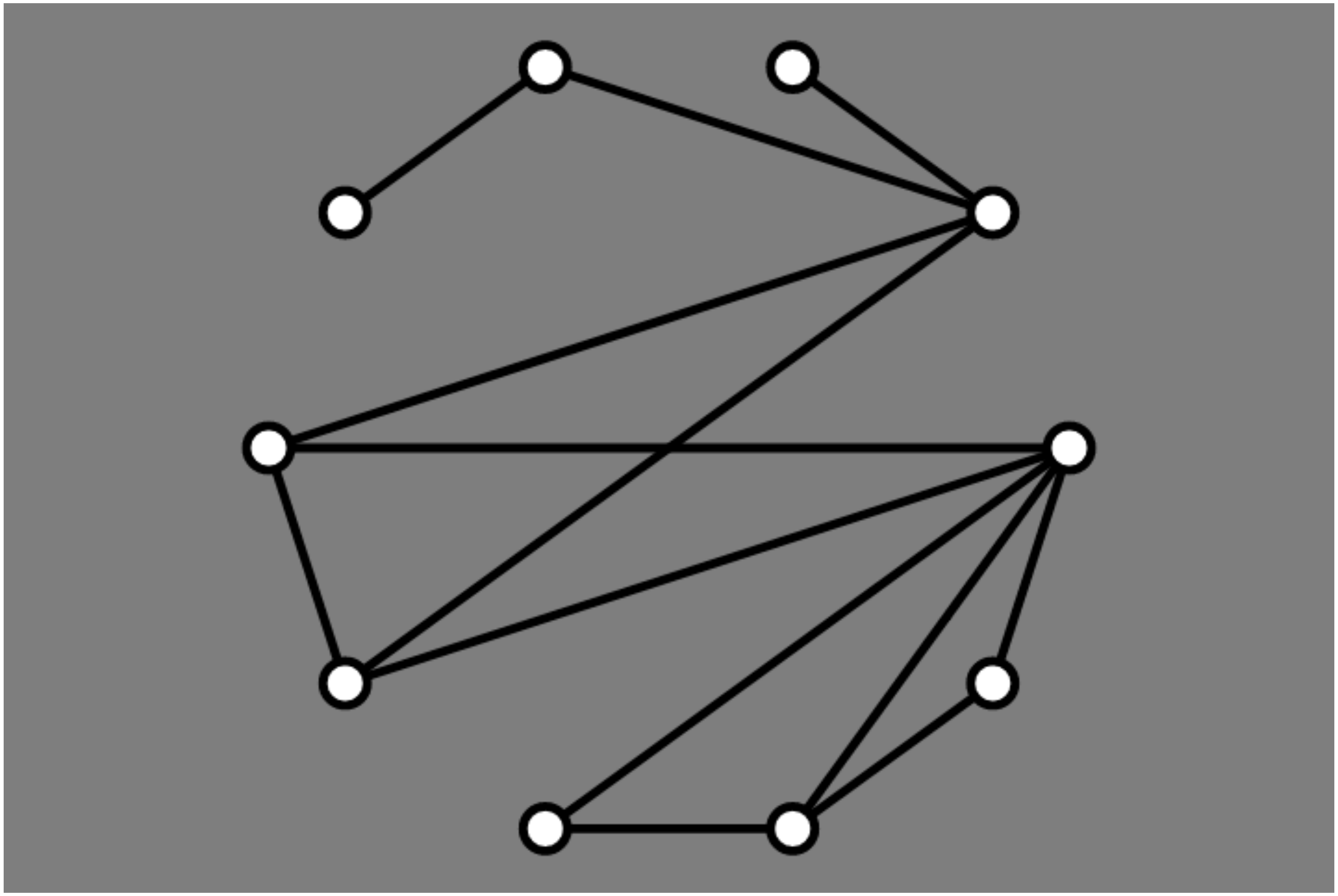

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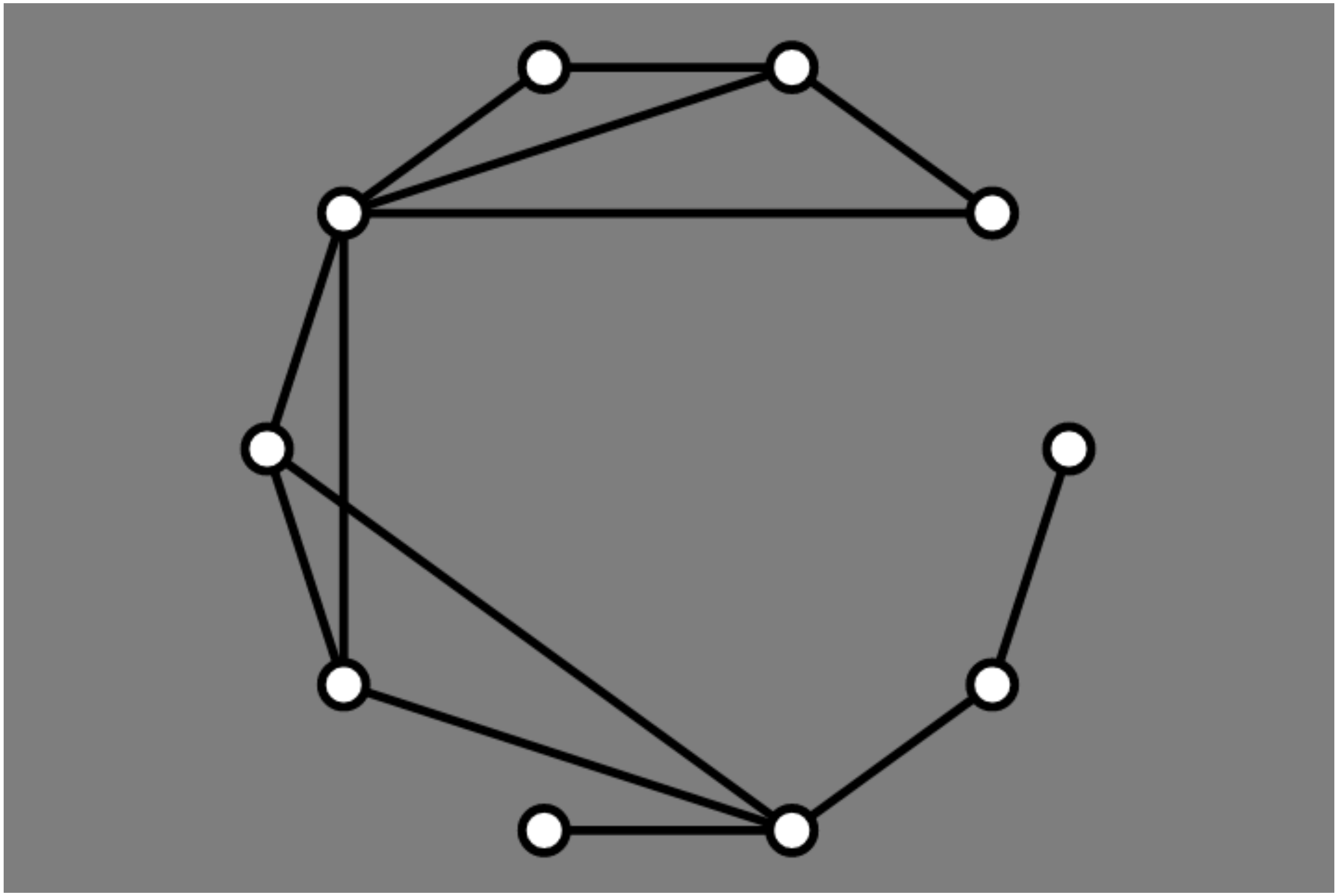
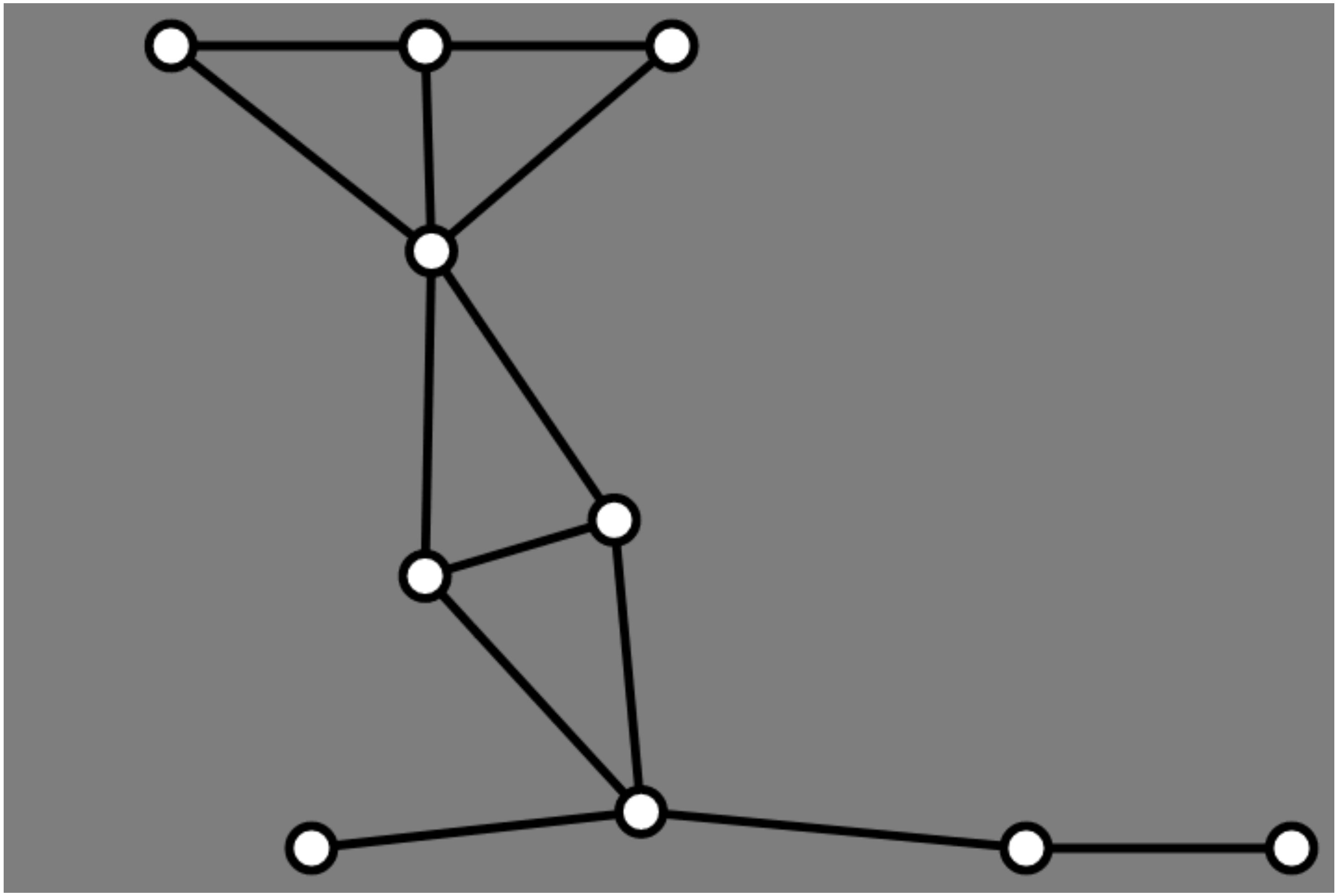


4: Force Directed



Okay

But it is **WAY BETTER** with animation

- Demo: `lec16-graph-drawing.zip`