Lecture 15: Drawing General Graphs I

COSC 225: Algorithms and Visualization

Spring, 2023

Announcements

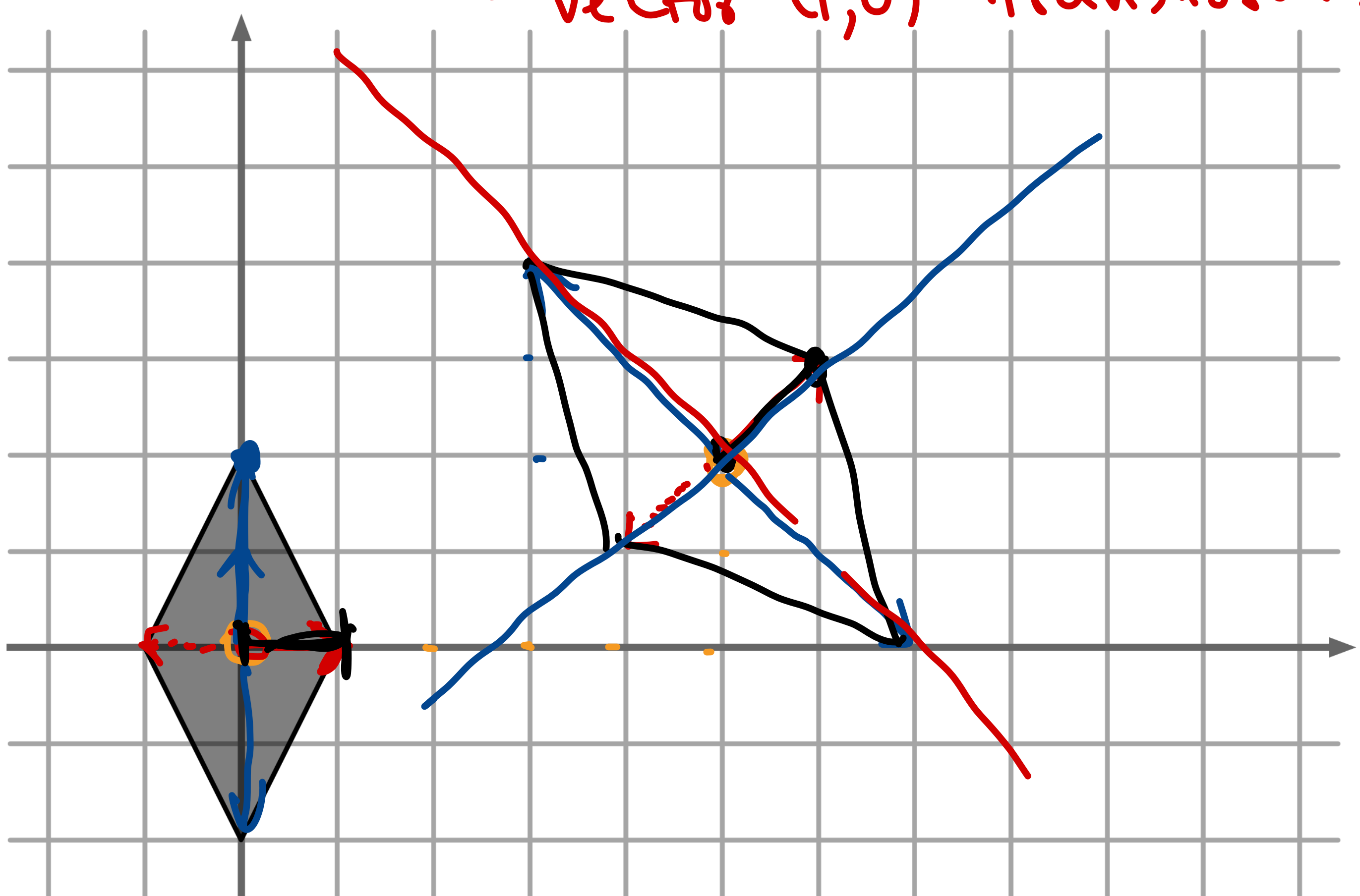
1. Assignment 07 posted, due Friday
2. Assignment 08 posted soon, due next Friday — last regular assgt
3. Final Projects
   - work with partner
   - topic: open-ended
   - requirement: build an interactive site with a significant algorithmic component
4. Limited OH This Week (advising week)
   - Short OH today
   - No OH on Thursday

Quiz 03, Question 1

vector (0,1) transforms

Apply `matrix(1, 1, −1, 1, 5, 2)` ← origin trans.

vector (1,0) transforms



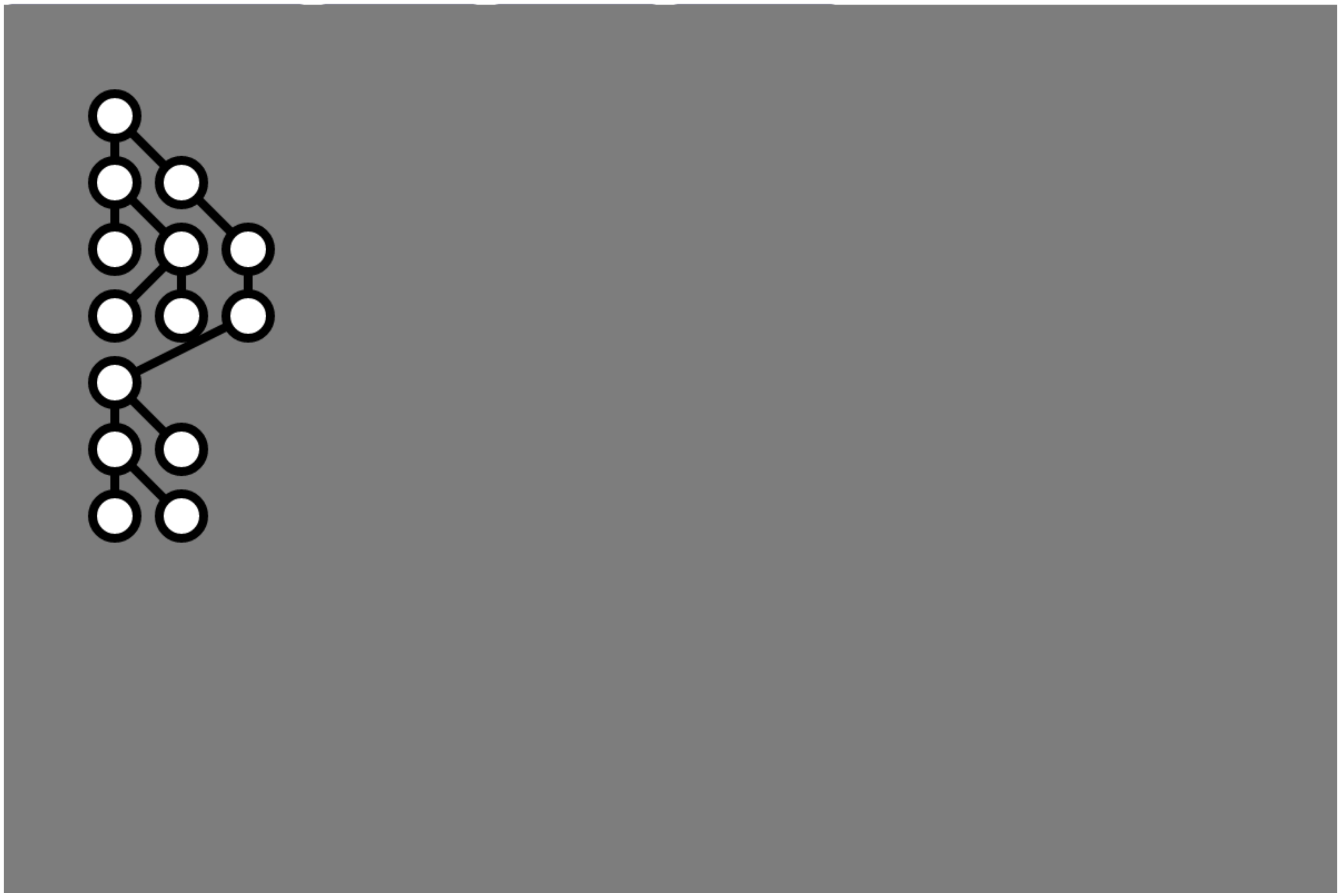
translate (5, 2) rotate (45 deg)
scale($\sqrt{2}$)

Outline

1. Drawing General Graphs
2. Circular Layouts
   - Mäkinen
   - AVSDF

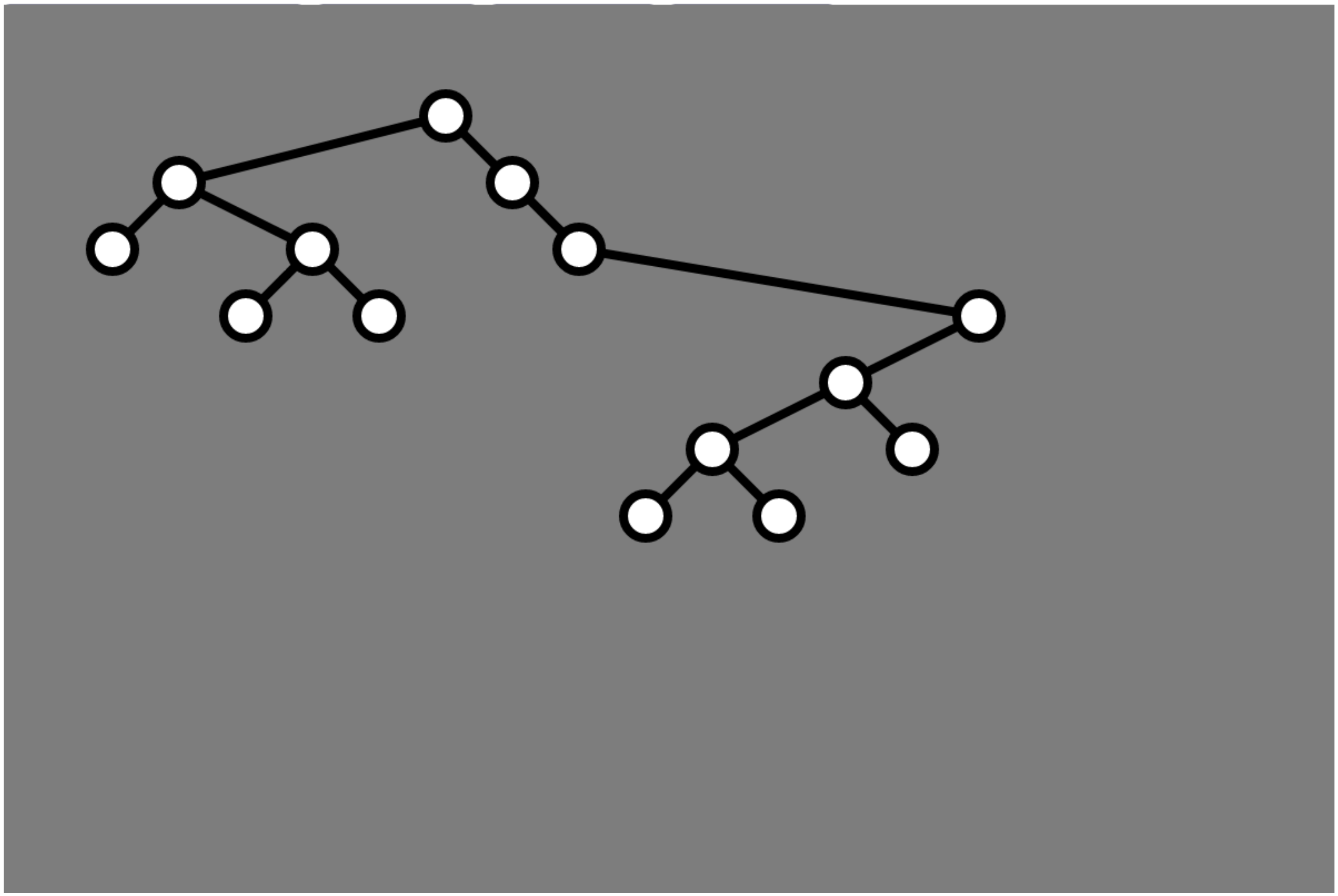
Last Time: Drawing Trees I

Greedy Layout



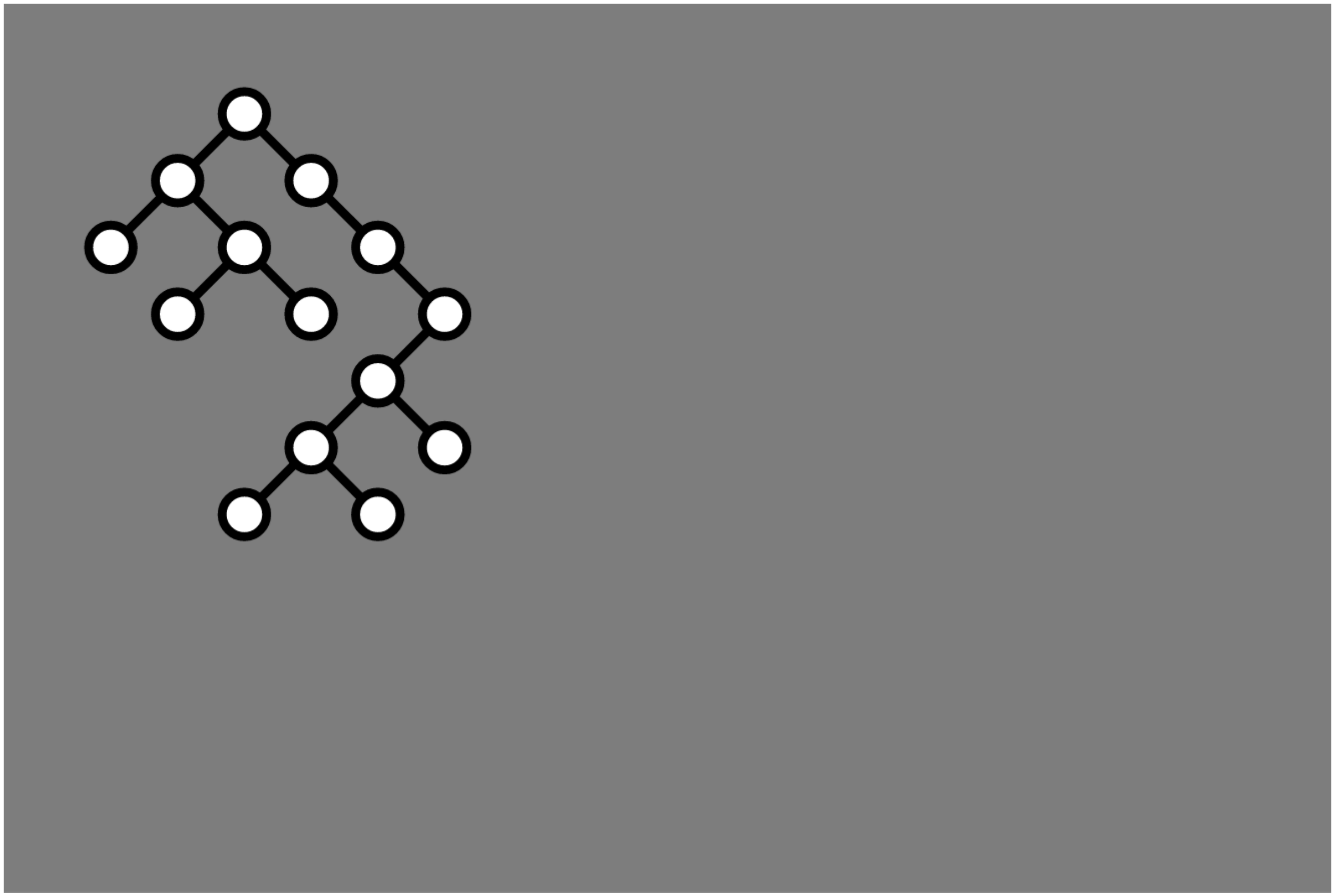
Last Time: Drawing Trees II

Knuth Layout



Last Time: Drawing Trees III

Tidy Layout



Today

Drawing general graphs

Input: set of vertices
edges between vertices

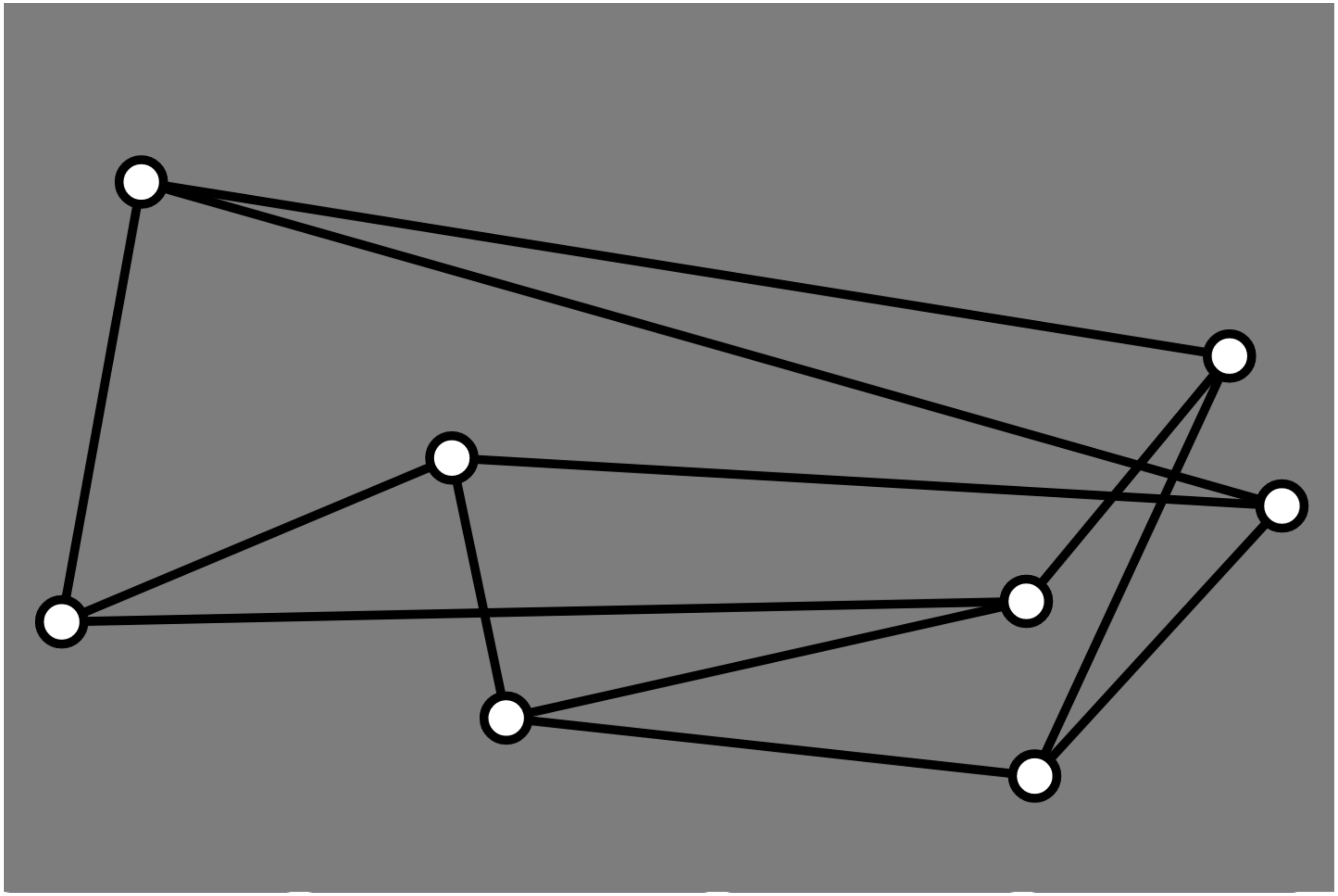
Output: positions for vertices
(how to draw edges too)

Warmup Activity

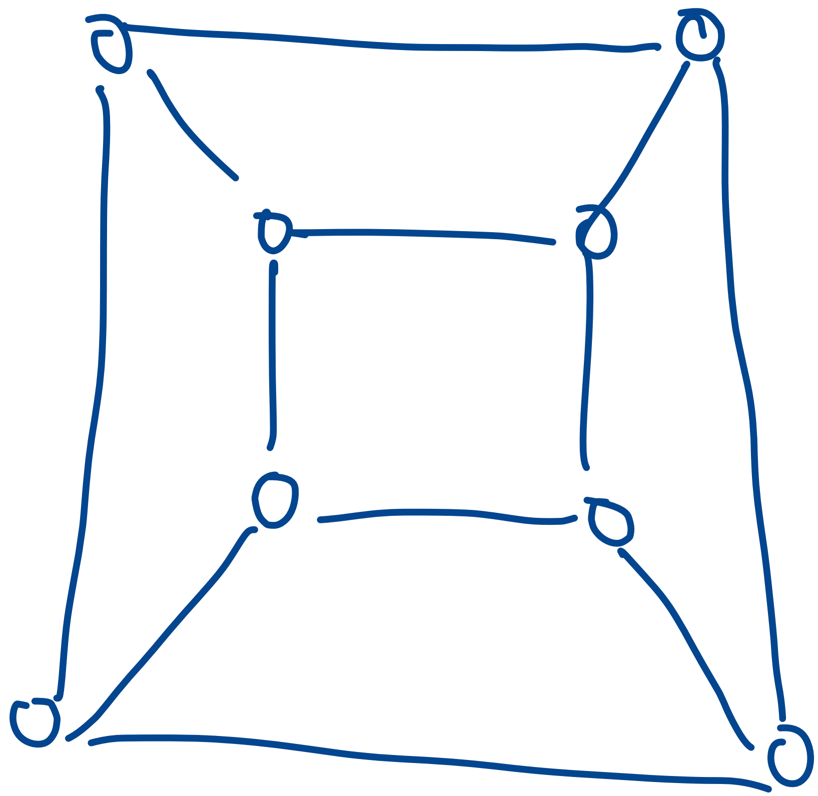
Draw a graph with the following adjacency lists

```
1: 6, 4, 7
2: 8, 5, 3
3: 6, 2, 4
4: 5, 3, 1
5: 4, 2, 7
6: 1, 3, 8
7: 5, 8, 1
8: 2, 6, 7
```

Random Layout



What Does Graph Look Like?



More Generally

What might we want in a graph layout?

- no crossing edges if possible?
- even lengths of edges
  - ~ vertices spread out
- ordering of vertices (categories)
- hierarchy

Desiderata

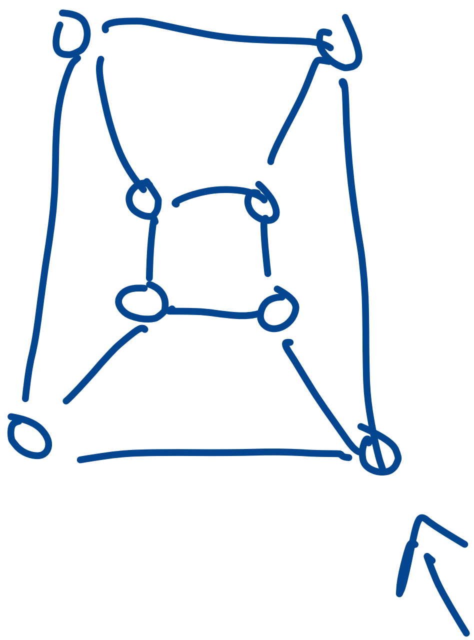
From Fruchterman and Reingold (1991):

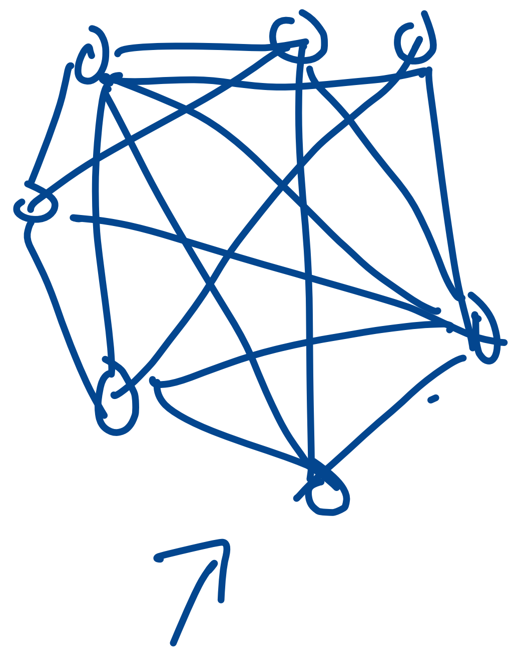
1. Distribute the vertices evenly in the frame.
2. Minimize edge crossings.
3. Make edge lengths uniform.
4. Reflect inherent symmetry.
5. Conform to the frame.

Interesting Question

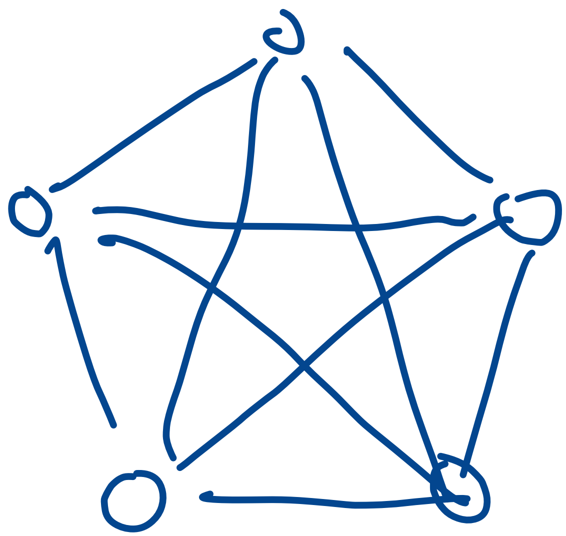
Which graphs can be drawn without any edge crossings?

- such graphs are called **planar graphs**

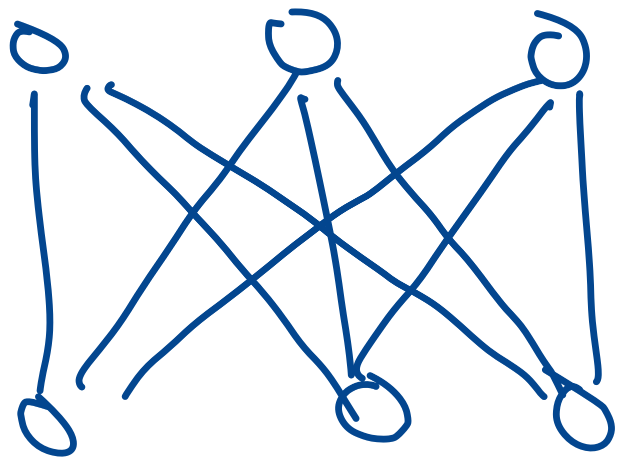




Minimal Non-planar Graphs



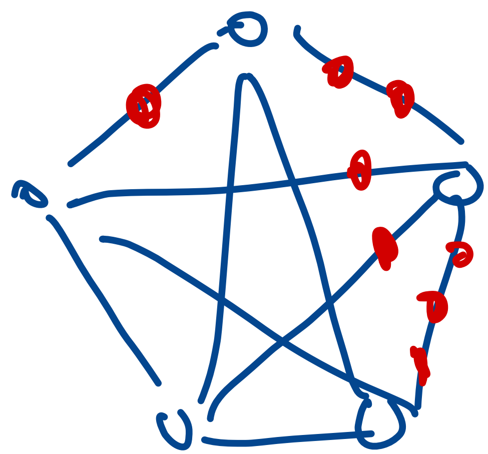
K_5



$K_{3,3}$

Characterization of Planar Graphs

A graph can be drawn w/out edge crossings if and only if it does not contain a subdivision of K_5 or $K_{3,3}$



Kuratowski 1930

Algorithmic Results

There are efficient algorithms for

1. detecting if a graph is planar
2. drawing a planar graph without edge crossings, e.g.:
    - Auslander-Parter 1961
    - Lempel-Even-Cederbaum 1967
    - ...

Implementing one would make an awesome final project!

Minimizing Edge Crossings

What about non-planar graphs? (Most graphs are not planar!)

Question. Can we efficiently draw graphs so as to minimize the number of edge crossings?

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Question. Can we efficiently draw graphs so as to minimize the number of edge crossings?

Answer. No!

- there is no known efficient algorithm for this task

Minimizing Edge Crossings

What about non-planar graphs? (Most graphs are not planar!)

Question. Can we efficiently draw graphs so as to minimize the number of edge crossings?

conjecture: cannot be solved efficiently

Answer. No!

- there is no known efficient algorithm for this task

More precisely. The following problem is NP-complete

- *Input:* A graph G and a natural number k
- *Output:* "yes" if G can be drawn with at most k crossings, and "no" otherwise

General Graph Drawing

Focus on *heuristics*

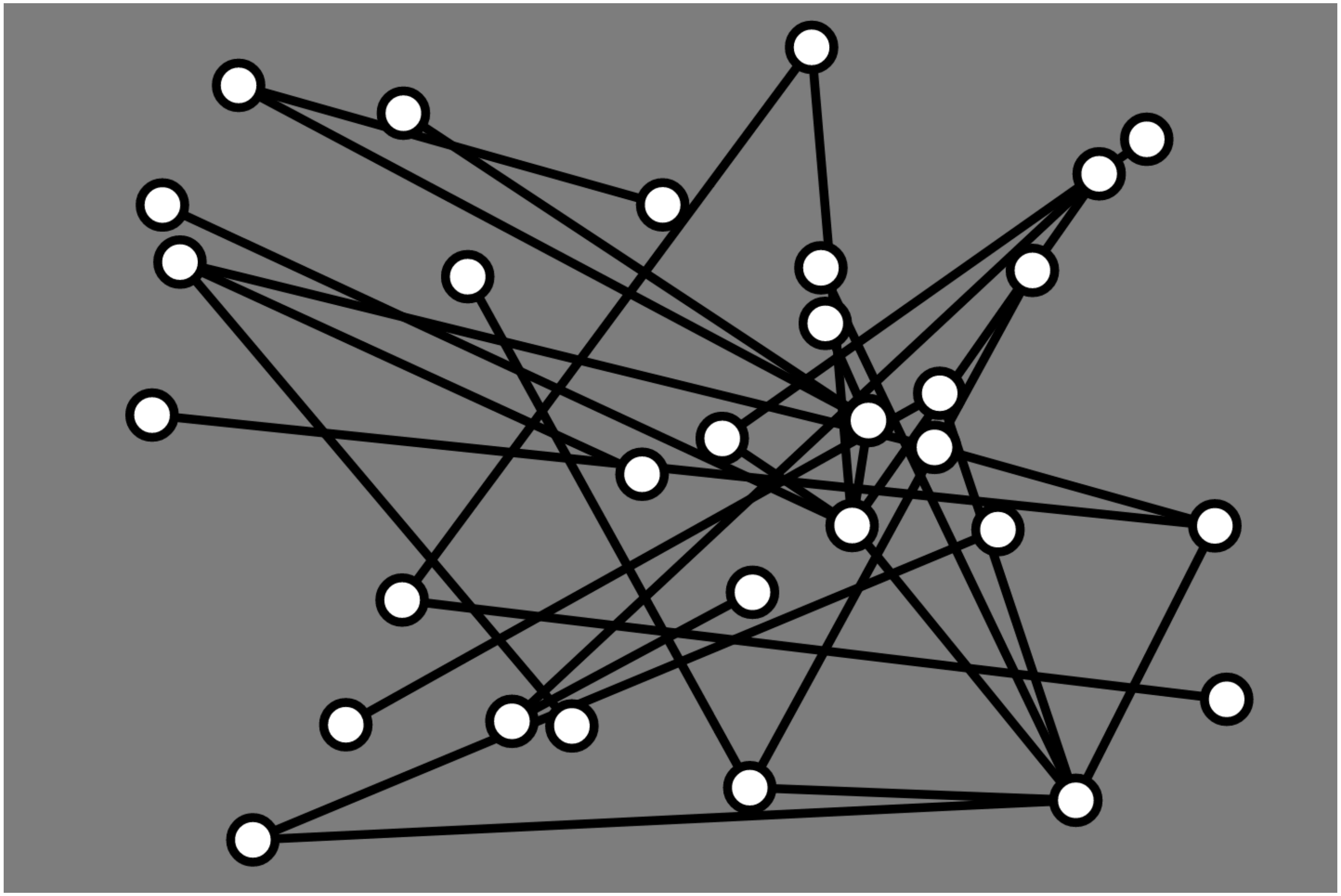
- do not *guarantee* that output minimized edge crossings, etc
- nonetheless have reasonably good results for the graphs we care about
- typically “simple” procedures

Two (of many) Approaches

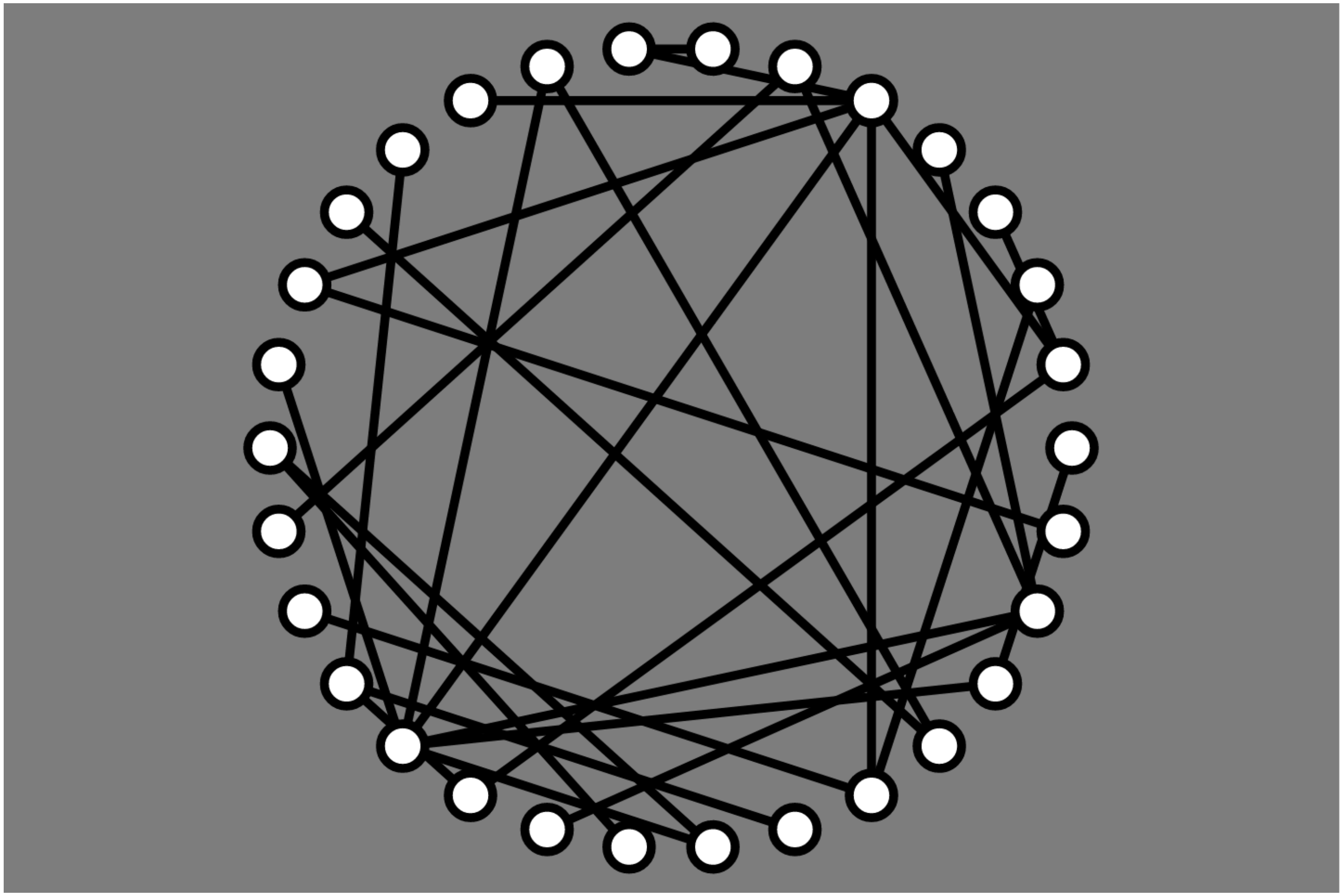
1. Circular Layouts (today)
   - fix vertices lie on a circle
   - pick an ordering of vertices to illustrate some graph features
2. Physical simulation layouts (Wednesday)
   - force-directed graphs
     - adjacent vertices attract each other (somewhat)
     - non-adjacent vertices repel each other
   - simulate physical system to determine vertex placement

Circular Layouts

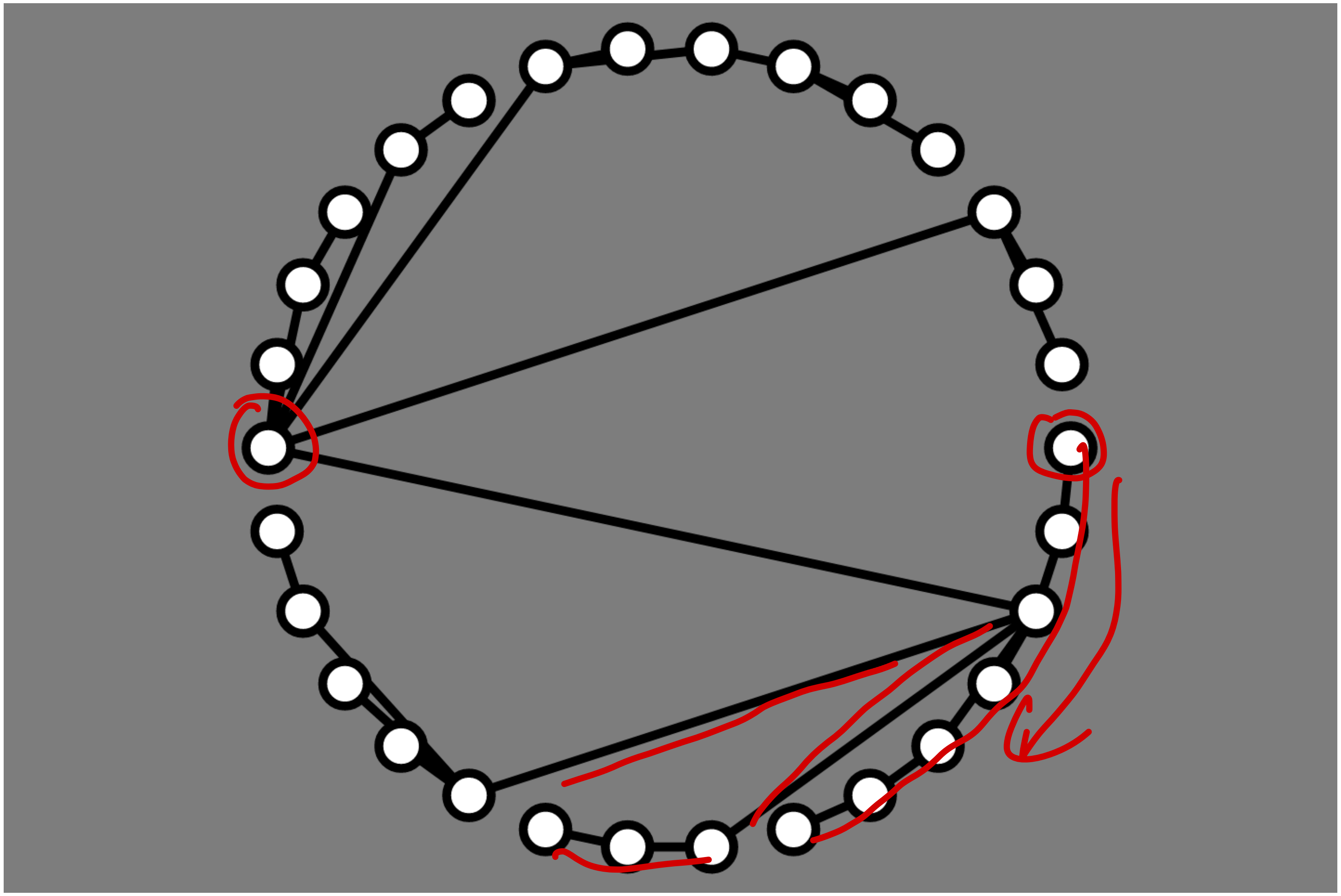
Progression I: Random



Progression II: Circular



Progression III: AVSDF



Adjacent Vertex Smallest Degree First

Starting Point

Framework from graph/DFS demos

- `Graph` object stores lists of vertices/edges
- `Vertex` object stores adjacency list (`neighbors`), has `x`, `y`
- `Edge` object represents a pair of vertices
- `GraphVisualizer` moderates interactions between site and `Graph` instance
  - draws vertices/edges
  - responds to user interactions

Adding Interactions

Previously:

- click to add vertices
- click pair of vertices to add edge

Added:

- hover to highlight a vertex and its neighbors
- demo: `lec15-graph-drawing.zip`

Implementing Hover Interactions

Added event listener to each vertex element

```
elt.addEventListener("mouseover", (e) => {
    this.muteAll();
    this.unmuteVertex(vtx);
    this.highlightVertex(vtx);
    for (let nbr of vtx.neighbors) {
        this.highlightVertex(nbr);
        this.highlightEdge(this.graph.getEdge(vtx, nbr));
    }
});
```

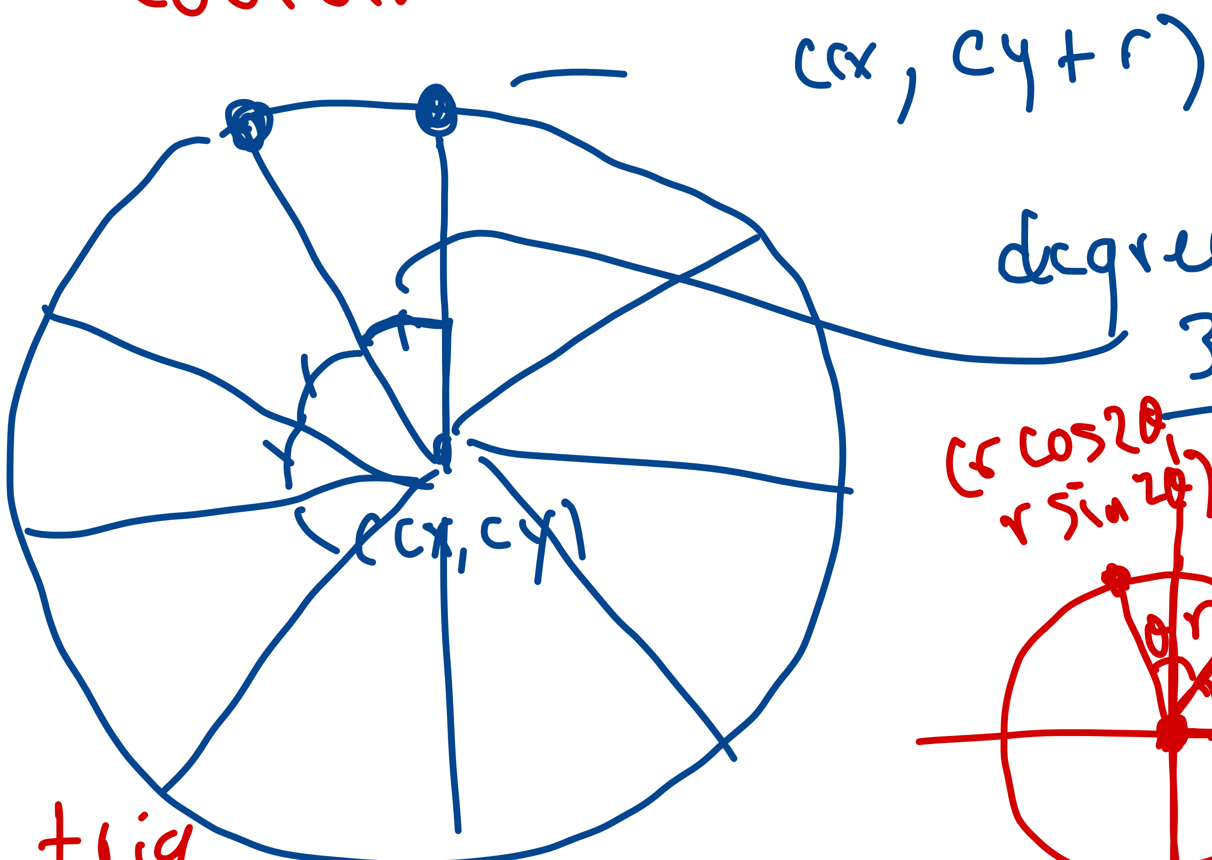
```
elt.addEventListener("mouseout", (e) => {
    this.unmuteAll();
    this.unhighlightAll();
});
```

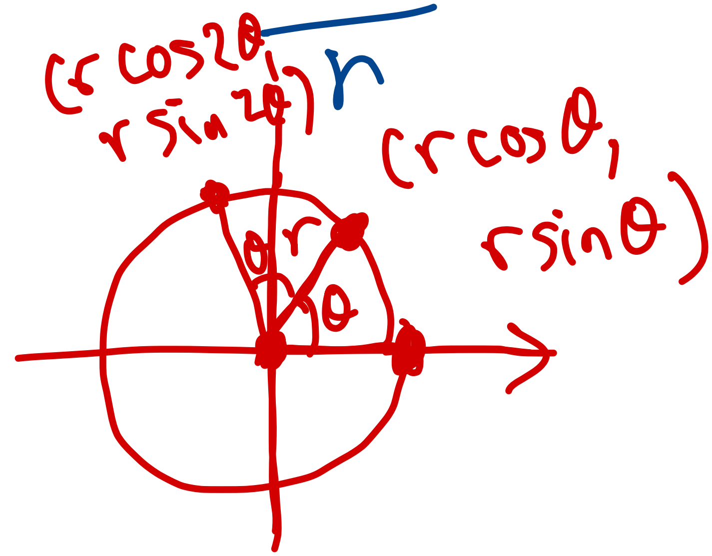
SVG element representing vertex vtx

Circular Embeddings

Setup: Graph with *n* vertices. How to set locations on a circle?

- radius = r
- coordinates of center (cx, cy)





do w/out trig
using transform?

Circular Embedding in Code

```
this.setLayoutCircle = function (cx, cy, r) {
let vertices = this.graph.vertices;
let n = vertices.length;
for (let i = 0; i < n; i++) {
  vertices[i].x = r * Math.cos(2 * Math.PI * i / n) + cx;
  vertices[i].y = r * Math.sin(2 * Math.PI * i / n) + cy;
}}
```

Simplified Problem

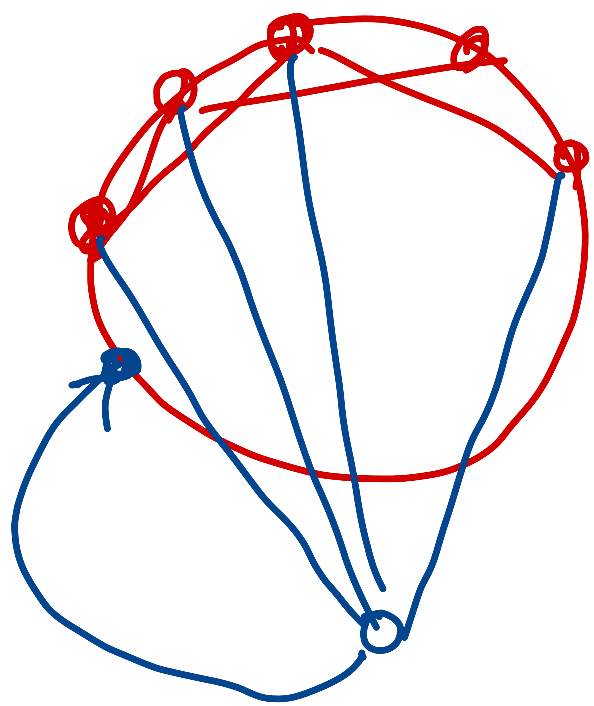
Now that we can draw vertices evenly around a circle, we can focus on the order in which to add vertices

- which ordering minimizes edge crossings?
  - no easier than general problem! ←
- which ordering is most informative?
- which ordering looks nice?

Mäkinen Heuristic

Basic idea:

- split vertices into `left` and `right` sets
- vertices with more `left` neighbors placed on `left` side
  - sim for `right` side

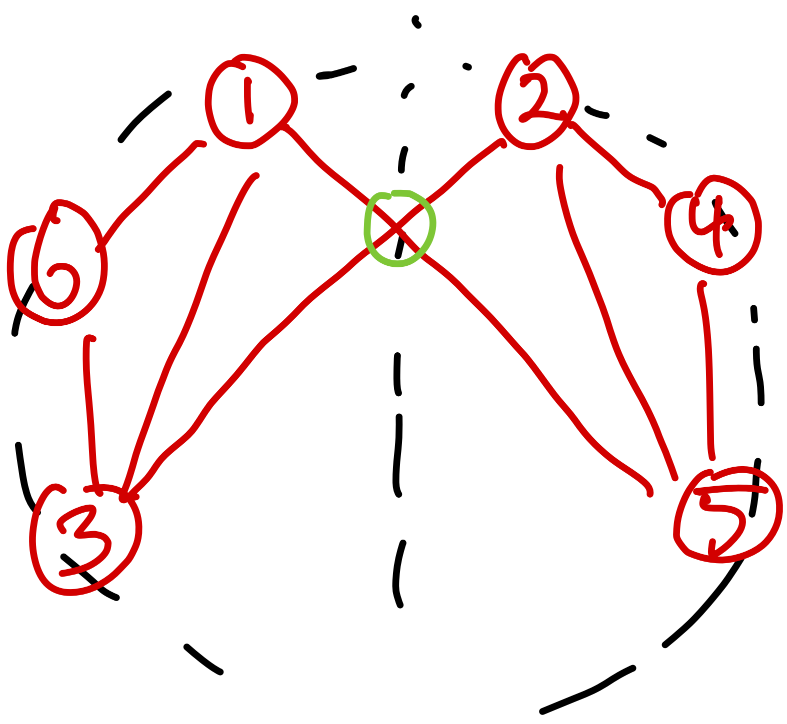


Mäkinen Procedure

1. Find two vertices of highest degree and add them to `left`/`right` sets
2. Repeat until all vertices are added to `left` or `right`:
   - compute (`right` neighbors) - (`left` neighbors) for each vertex
   - add vertex with largest value to `right`
   - add vertex with smallest value to `left`
3. Add `left` vertices on left side, `right` on right side

Mäkinen Example

```
1: 2, 6, 3, 5
2: 1, 3, 5, 6
3: 1, 2, 6
4: 2, 5
5: 1, 2, 4
6: 1, 3
```



How To Implement Mäkinen Efficiently

- What do we keep track of and store?
- How do we update data structures?
- How efficient is the procedure

Mäkinen Procedure, Again

1. Find two vertices of highest degree and add them to `left`/`right` sets
2. Repeat until all vertices are added to `left` or `right`:
   - compute (`right` neighbors) - (`left` neighbors) for each vertex
   - add vertex with largest value to `right`
   - add vertex with smallest value to `left`
3. Add `left` vertices on left side, `right` on right side

Data Structures

```
const vertices = this.graph.vertices;
const n = vertices.length;
const leftPlaced = [];
const rightPlaced = [];
const placed = new Array(n).fill(false);
const leftCount = new Array(n).fill(0);
const rightCount = new Array(n).fill(0);
let placedCount = 2;
```

already places

Initialization

```
vertices.sort((u, v) => {
    return u.degree() - v.degree();
});

// two highest degree vertices go on left and right sides
let left = vertices[n-1];
leftPlaced.push(left);
placed[left.id] = true;
let right = vertices[n-2];
rightPlaced.push(right);
placed[right.id] = true;
```

Main Loop I

```
for (let vtx of left.neighbors) { leftCount[vtx.id]++; }
for (let vtx of right.neighbors) { rightCount[vtx.id]++; }

for (let vtx of vertices) {
  if (!placed[vtx.id]) {
    left = vtx;
    right = vtx;
    break;
  }}
```

newly placed vertices

find an unplaced vertex

Main Loop II

```
// set right and left to be the vertices maximizing and
// minimizing (respectively) the quantity rightCount -
// leftCount
for (let vtx of vertices) {
  if (/* most right - left nbrs */) {
    right = vtx;
  }

  if (/* least right - left nbrs  */) {
    left = vtx;
  }}
```

What is Overall Running Time?

Assume graph has n vertices, m

See Demo

AVSDF Heuristic

Adjacent **V**ertex **S**mallest **D**egree **F**irst

- He & Sykora

Idea:

- perform depth-first search, starting from vertex of minimal degree
- always explore minimum degree neighbor first

AVSDF Example

```
1: 2, 6, 3, 5
2: 1, 3, 5, 6
3: 1, 2, 6
4: 2, 5
5: 1, 2, 4
6: 1, 3
```

How To Implement AVSDF Efficiently

- What do we keep track of and store?
- How do we update data structures?
- How efficient is the procedure

AVSDF Initialization

```
const order = [];
const stack = [];
const vertices = this.graph.vertices;
const n = vertices.length;
const placed = new Array(n).fill(false);

vertices.sort((u, v) => {
    return u.degree() - v.degree();
});

stack.push(vertices[0]);
```

Main Loop

```
while (stack.length > 0) {
  let vtx = stack.pop();
  if (!placed[vtx.id]) {
    order.push(vtx);
    placed[vtx.id] = true;
    vtx.neighbors.sort((u, v) => {
      return v.degree() - u.degree();
    });
    for (let nbr of vtx.neighbors) {
      if (!placed[nbr.id]) { stack.push(nbr); }
    }}}
```

Running Time of Main Loop?

When Will Algorithm Fail?

AVSDF Demo

Next Time

Force-directed layout