Lecture 13: Drawing Binary Trees I

COSC 225: Algorithms and Visualization

Spring, 2023

Annoucements

48 hr extension is fine

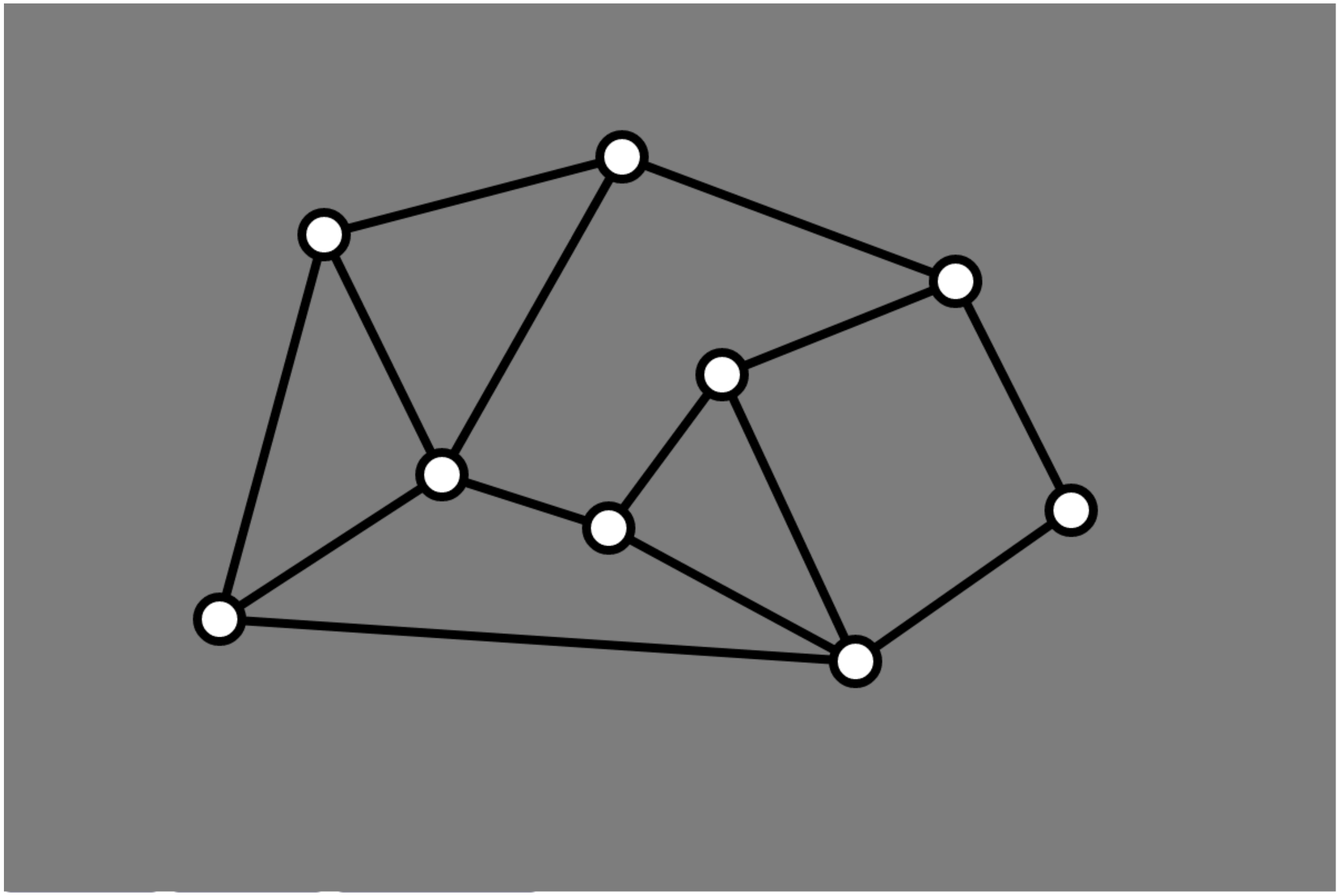
- Assignment 06 due tonight
- Assignment 07 posted soon
  - make a site that incorporates recursion and coordinate transformation to make a self-similar image
  - due next Monday
- Quiz this Wednesday: coordinate transformations
  - define a `matrix` transformation given transformation's geometric description
  - given a `matrix` and original image, draw the transformed image

Outline

1. Binary Trees
2. Activity: Drawing Binary Trees by Hand
3. Aesthetic and Pragmatic Principles
4. Greedy Procedure
5. Knuth Layout

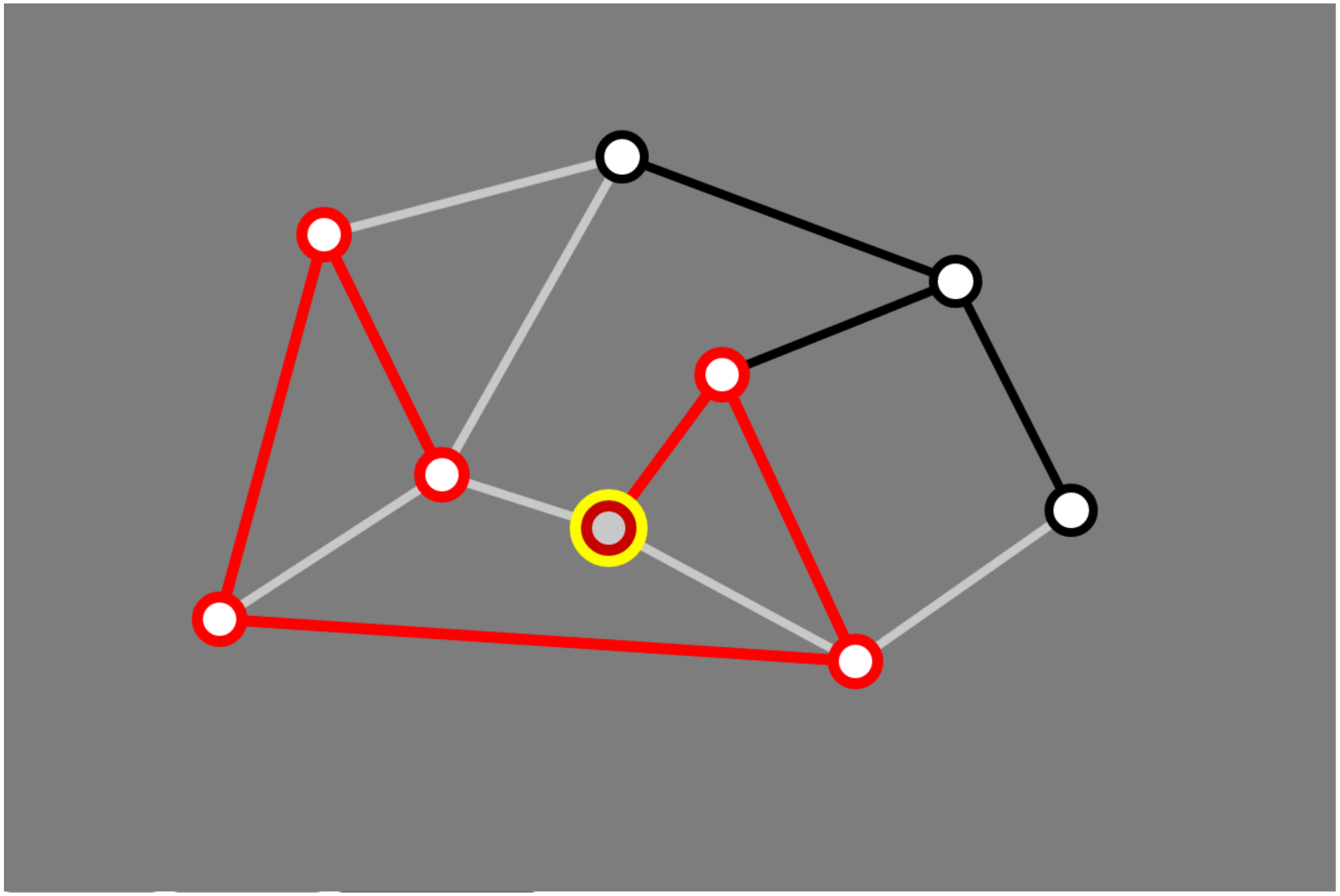
A While Back

We illustrated the depth-first search algorithm on graphs



A While Back

We illustrated the depth-first search algorithm on graphs



Interaction

User drew input graph graph by hand

- from clicks, obtained graph structure
- geometry of graph layout defined with graph

Graph Drawing

Input. A graph

- vertices
- edges

Output. A drawing of the graph

- visual representation of vertices
    - geometric locations
    - usually “points”
- visual representation of edges
    - usual lines or curves between vertices

This Week

Algorithms for drawing **binary trees**

Recall

A (rooted) **binary tree** consists of

- a set V of vertices
- a **root** vertex
- each vertex has:
  - a **left child** (possibly null)
  - a **right child** (possibly null)

satisfying:

- the root is not anyone's child
- every node is the child of exactly one node
- every node is a descendant of the root

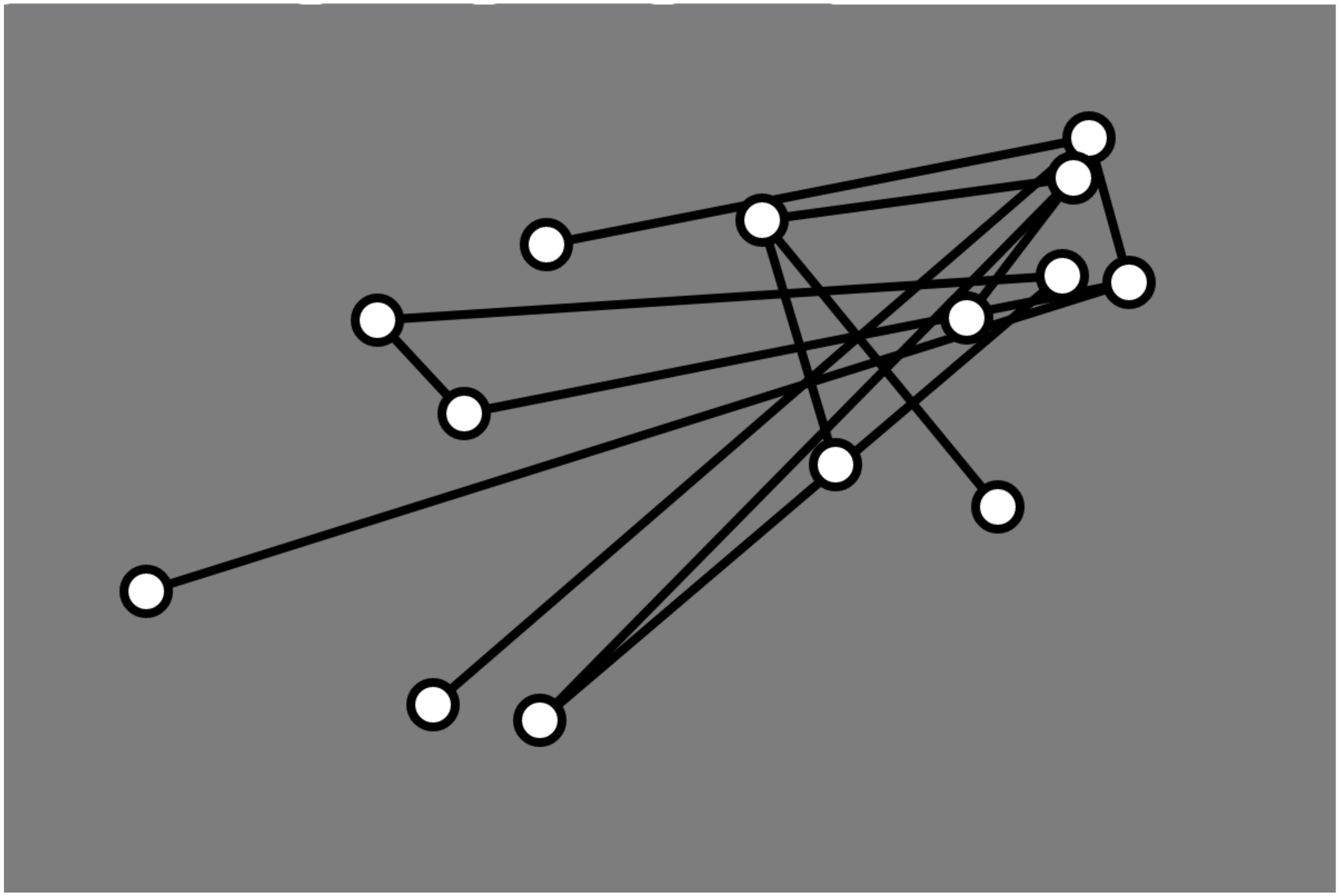
Example

- `V = {0, 1, 2, 3, 4}` vertices
- `root = 0`
- `left(0) = 2, right(0) = 3`
- `left(3) = 4, right(3) = 1`
- unassigned children are `null`

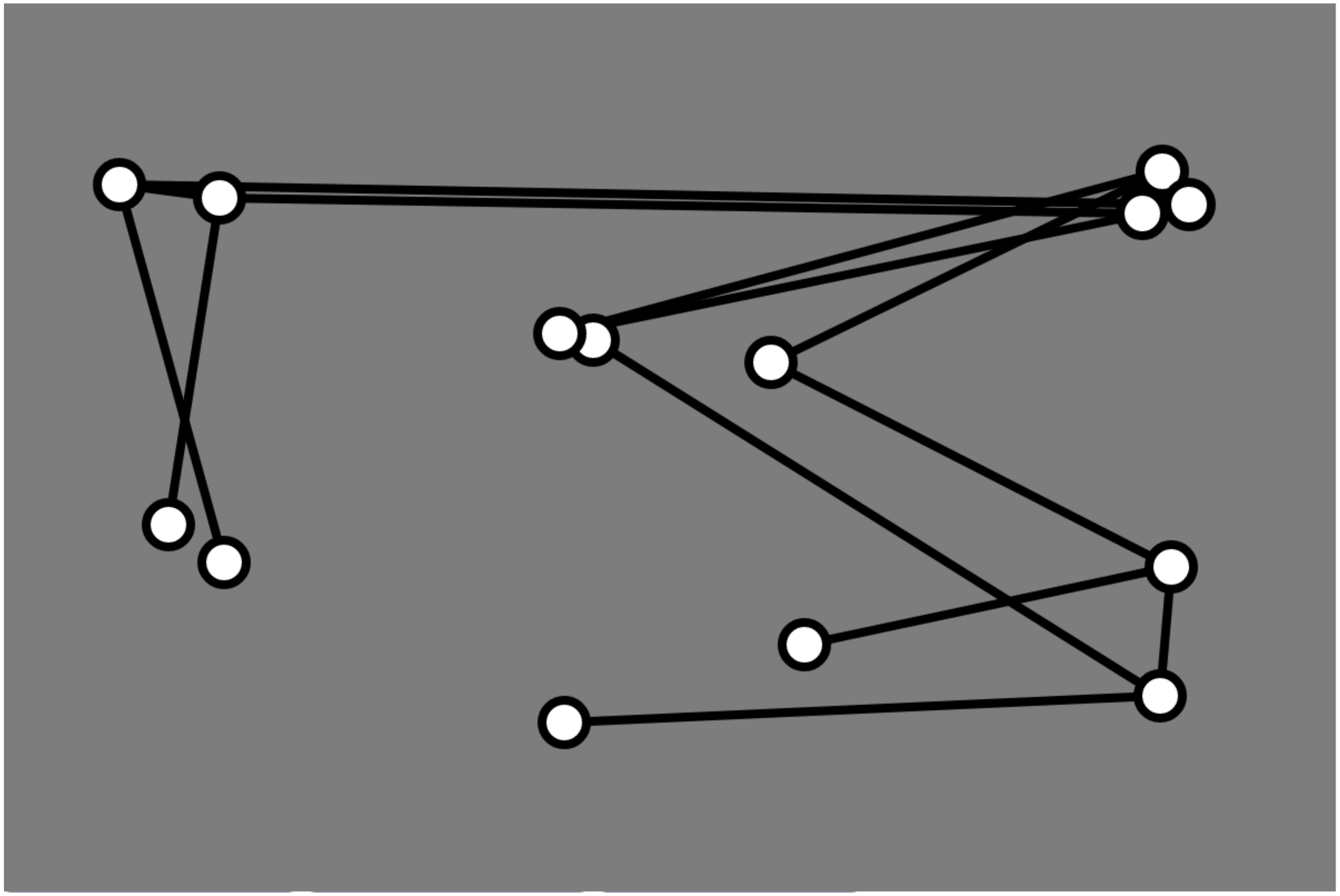
Activity: Draw a Tree

```
V = {0,...,13}
root: 0
left(0) = 1, right(0) = 2
left(1) = 3, right(1) = 4
right(2) = 5
left(4) = 6, right(4) = 7
right(5) = 8
left(8) = 9
left(9) = 10,  right(9) = 11
left(10) = 12, right(10) = 13
```

You Drew This, Right?

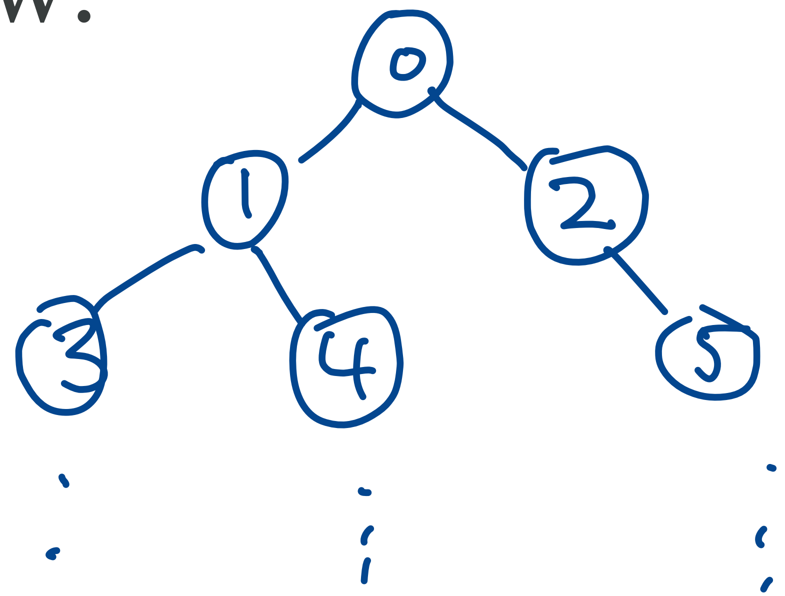


How About This?



What Did You Draw?

- Root @ top
- left children go left
- right children go right
- deep (farther from root) is downward



Questions

1. What information do we want to convey about the tree?
2. What constraints might we have on our drawing?
3. What aesthetic considerations might we have?
    - when does a tree “look nice?”

What Information Should the Drawing Convey?

- parent child relationship
  - parents above / children below
- no overlap @ same level
  - vertical position ~ depth
- distinguish left/right children
- whole tree visible

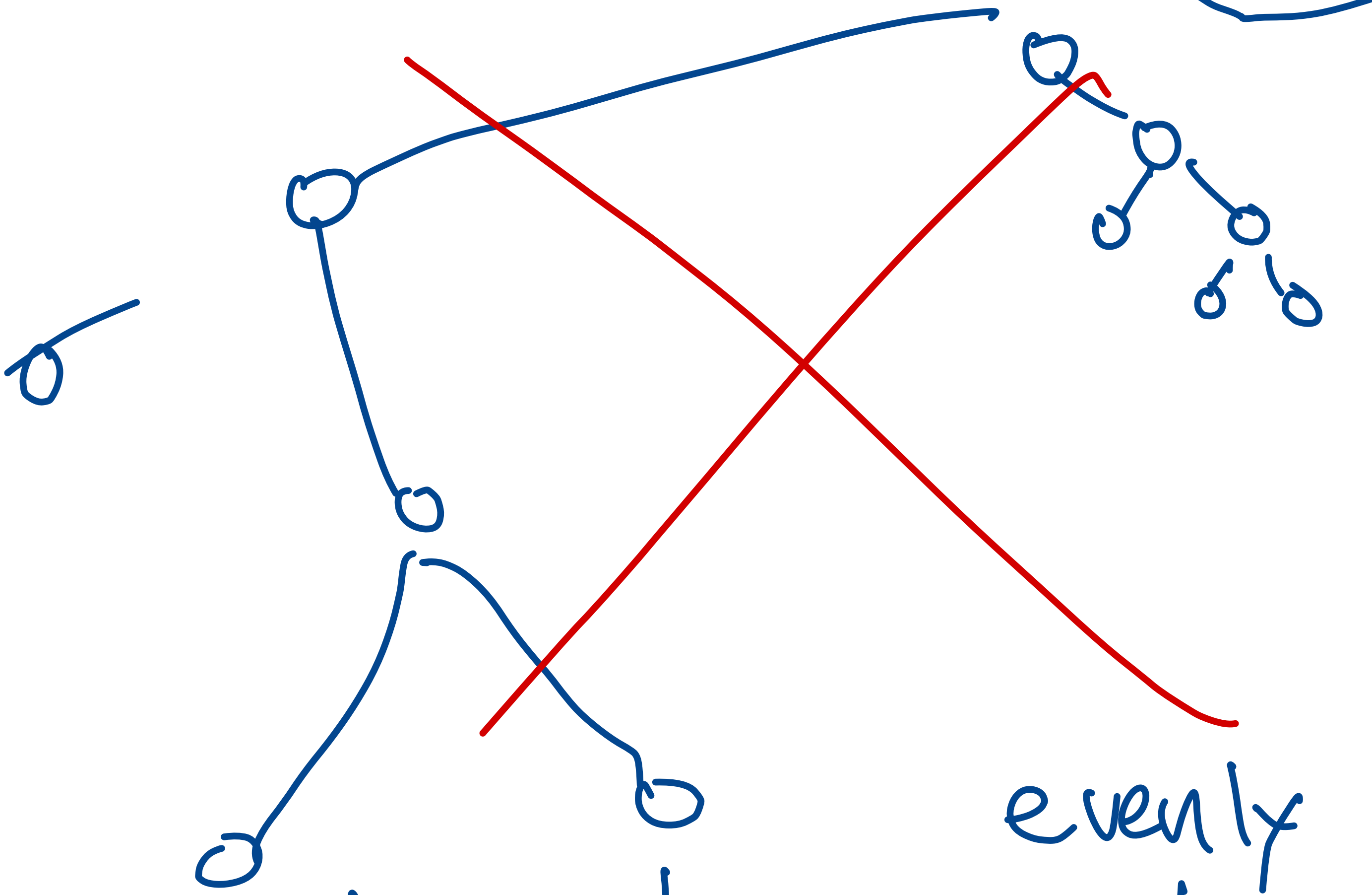
What Constraints Should we Consider?

- viewport has fixed height/width
- minimum size/separation between vertices

Aesthetic Considerations?

large enough components to convey data

- balance / symmetry



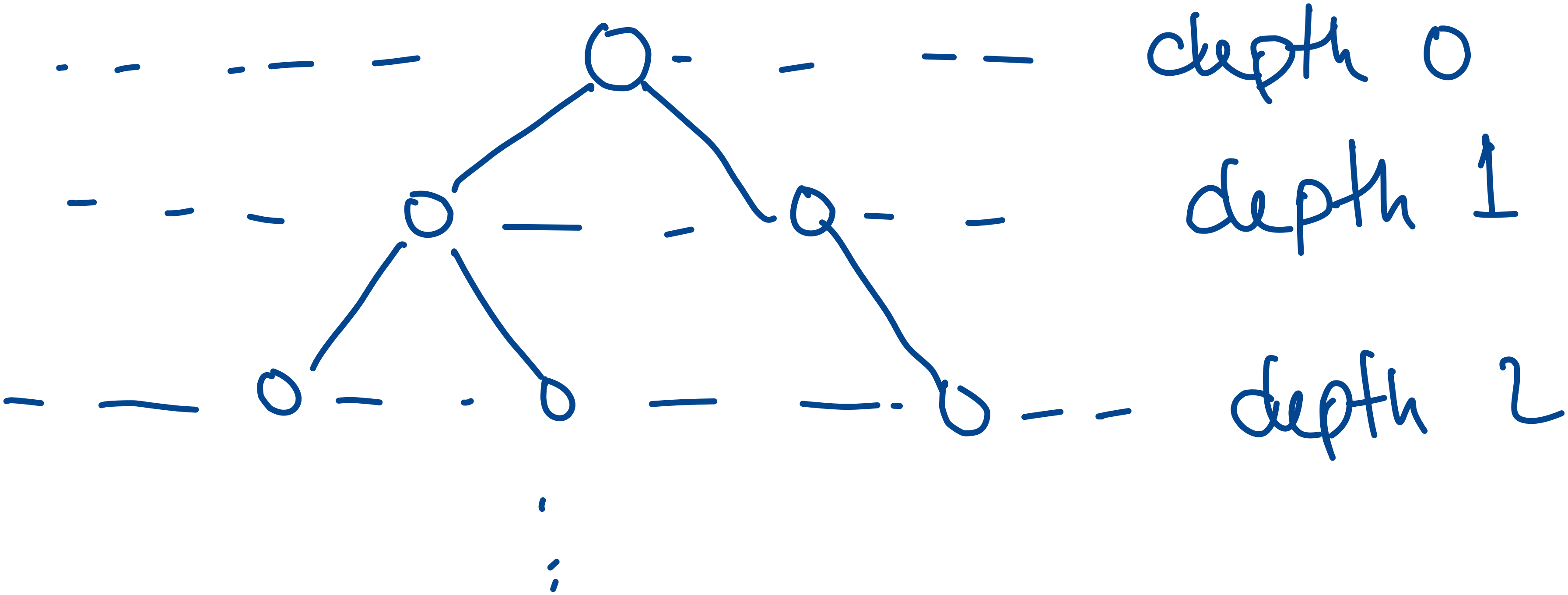
- Grid layout

evenly spaced vertices

First Principle

Aesthetic Principle 1. Vertices at the same **depth** should lie along a horizontal line with deeper nodes lower than shallower nodes.

- what physical requirements does this impose?



Height of drawing now depends linearly on depth of tree

Physical Limitation

Have to contend with width

- What can we do about it?

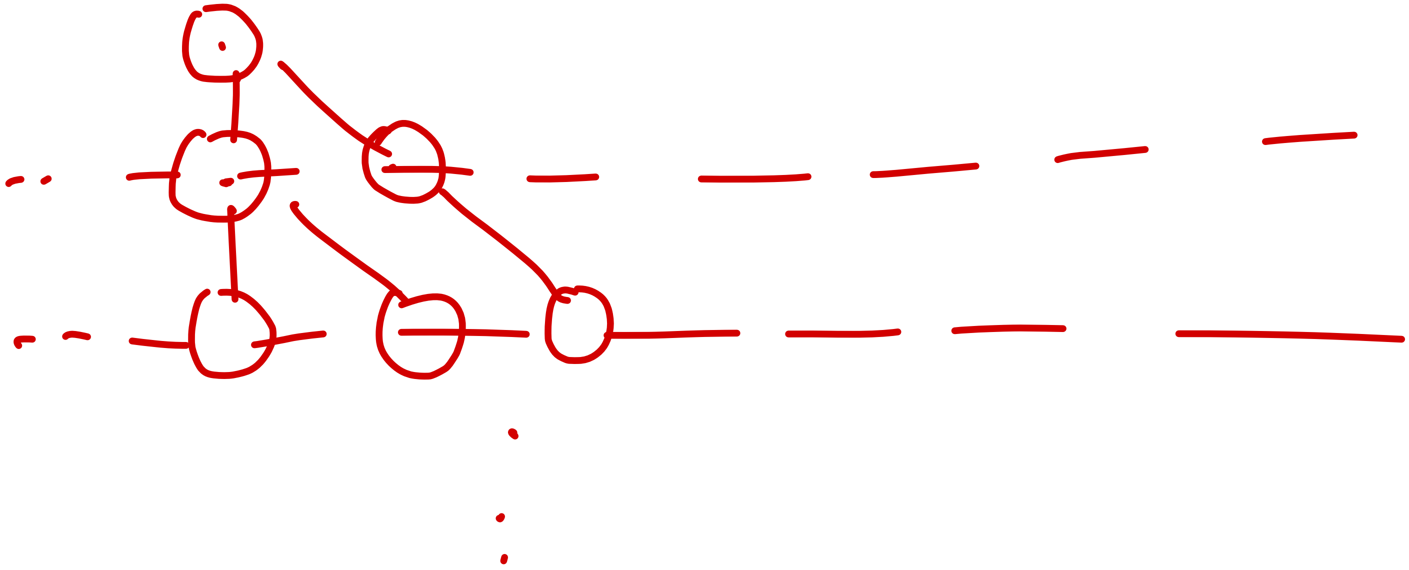
– scrolling?

– partition horizontal space
w/ roof @ center?

Optimal Layout?

How can we achieve minimum possible **width** subject to

1. lower bound on horizontal spacing
2. Aesthetic Principle 1



Greedy Layout

Idea

- draw vertices in rows according to **depth**
  - depth = distance from root
- root goes alone in the top row, next row at depth 1, etc.
- draw each row with vertices from "left to right"
  - what does this mean?

↪ left child always to left of right child

Greedy Layout

Idea

- draw vertices in rows according to **depth**
  - depth = distance from root
- root goes alone in the top row, next row at depth 1, etc.
- draw each row with vertices from “left to right”
  - what does this mean?

Promise

- Use as few columns as possible!
  - minimize width requirement

Greedy Layout Illustrated

V = {0,...,7}

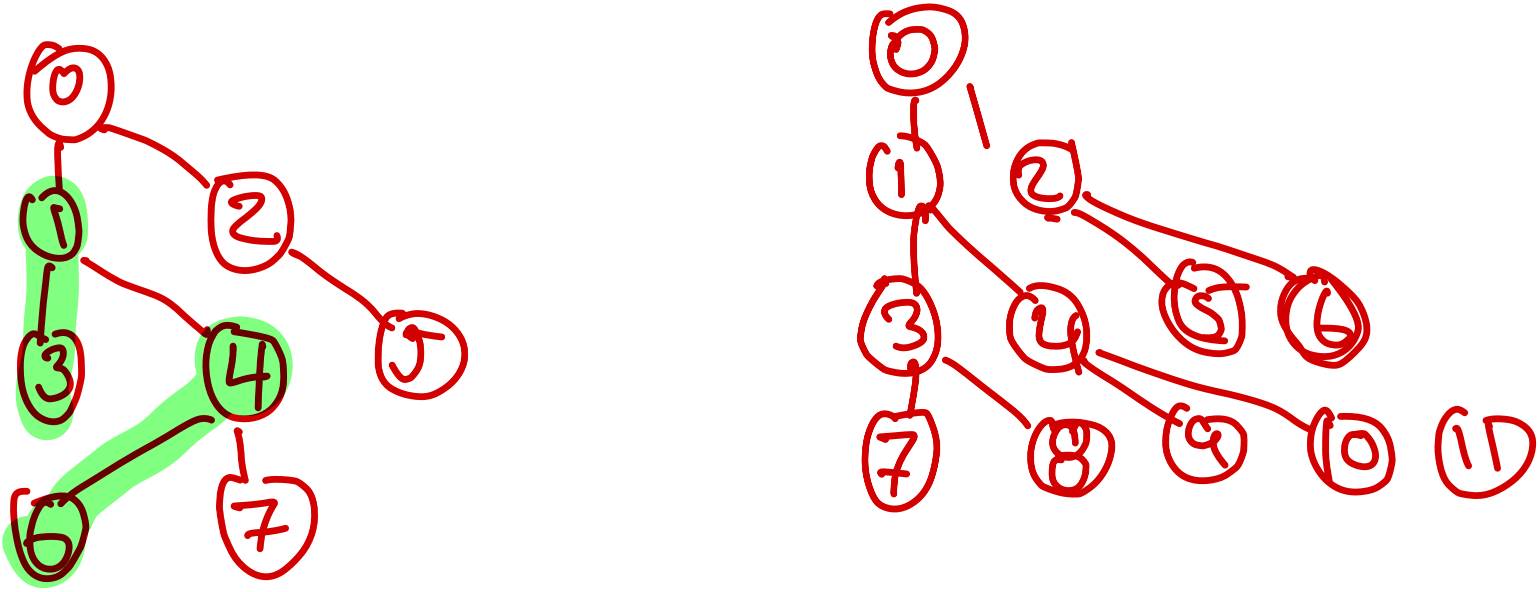
root: 0

left(0) = 1, right(0) = 2

left(1) = 3, right(1) = 4

~~right(2) = 5~~ left(2) = 5

left(4) = 6, right(4) = 7



How to Implement Greedy Layout?

Input: tree (just the root?)

Output: row and column for each ~~node~~ vertex

= depth ← compute this!

- max # of nodes a depth

↪ how many to left of each vertex?

How to Get Depths of Nodes?

traverse from node to parent
→ continue to root,
count how far

set root depth 0
go to children (if any)
update their depth to
1 + parent depth

How to Get Depths of Nodes?

My implementation:

- set depth when each vertex is added
- depth of a vertex is parent’s depth + 1
- store a `Map`:
  - keys are vertex IDs
  - values are depths

How to Get Columns?

For each depth d, keep track of left-most un-occupied column

Traverse the tree
- when visiting a ~~node~~ vertex put vertex @ left most un occupied col at its depth, increment col # for that depth

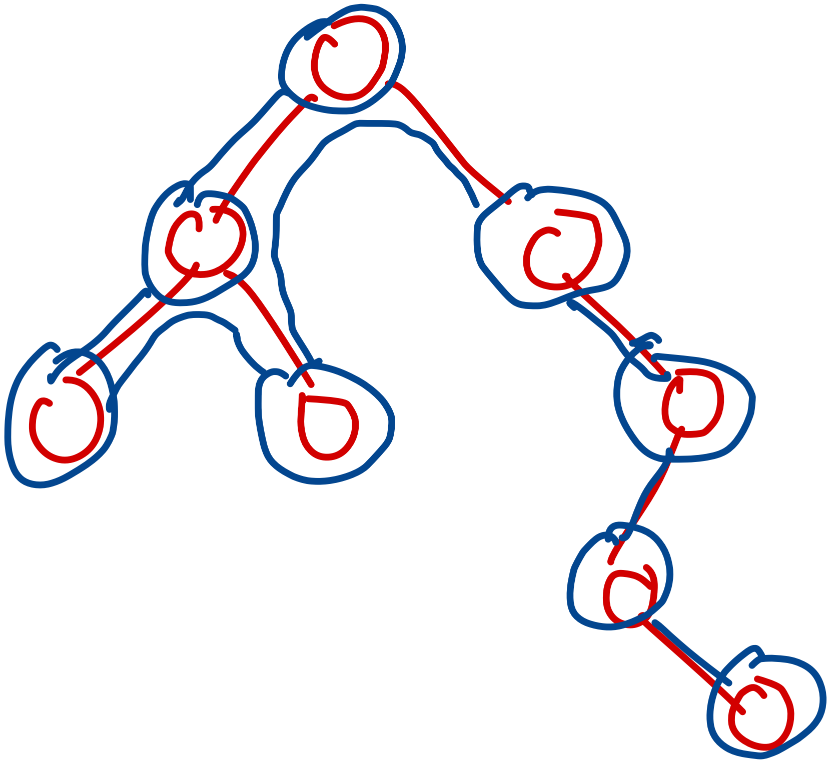
How to Get Columns?

Observation. If u is a left child of v and w is a right child of v, then u should be in a column to the left of v.

Idea. Starting from the root:

1. place vertex in the left-most un-assigned column in its row (depth)
2. place **left** descendants
3. place **right** descendants

This is *pre-order traversal!*

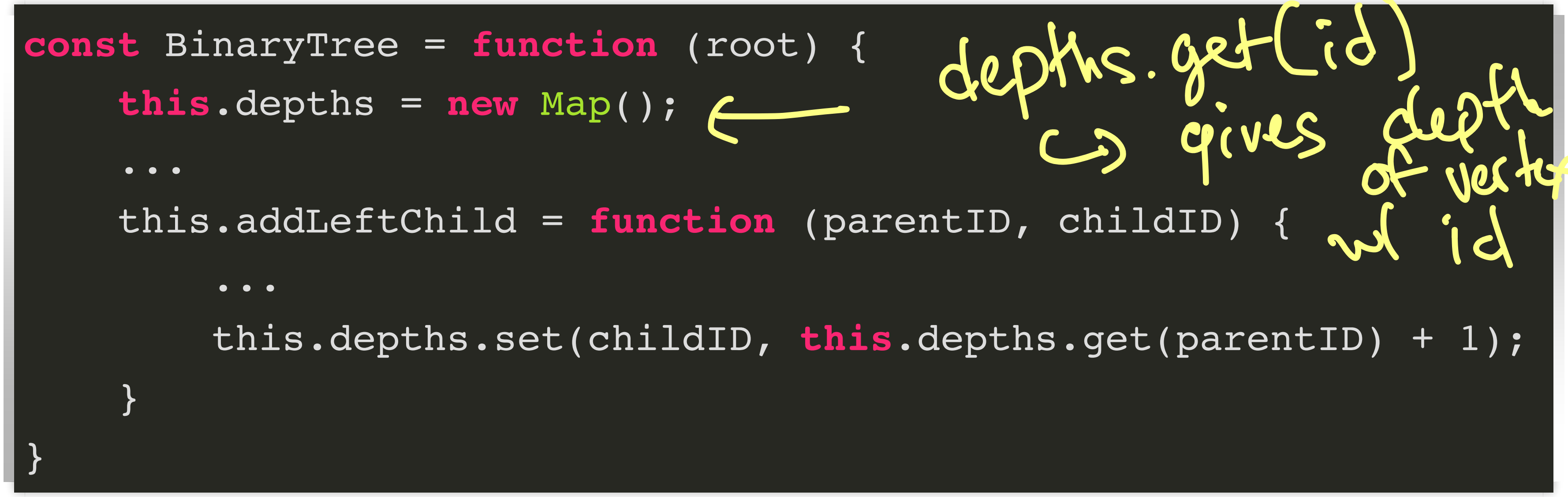


Column Assignment Illustrated

```
V = {0,...,7}
root: 0
left(0) = 1, right(0) = 2
left(1) = 3, right(1) = 4
right(2) = 5
left(4) = 6, right(4) = 7
```

Greedy Layout in JavaScript

Computing Depths:



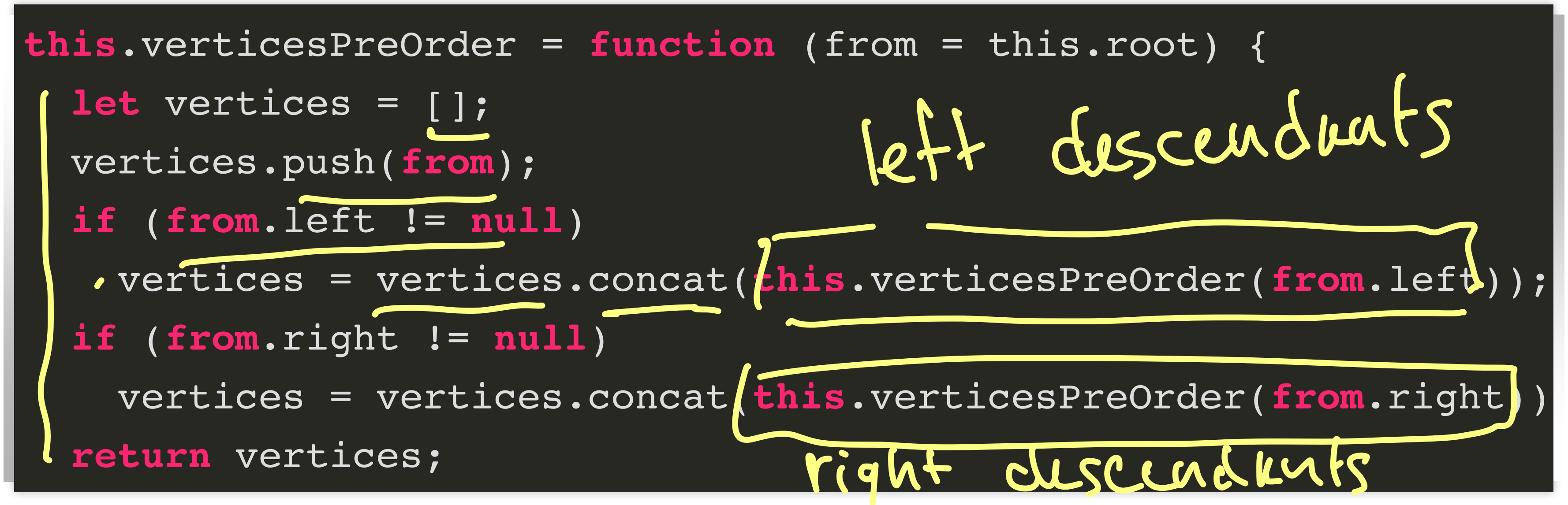
```
const BinaryTree = function (root) {
    this.depths = new Map();
    ...
    this.addLeftChild = function (parentID, childID) {
        ...
        this.depths.set(childID, this.depths.get(parentID) + 1);
    }
}
```

←

depths.get(id)
↳ gives depth of vertex w/ id

Greedy Layout in JavaScript

Getting vertices in “pre-order”



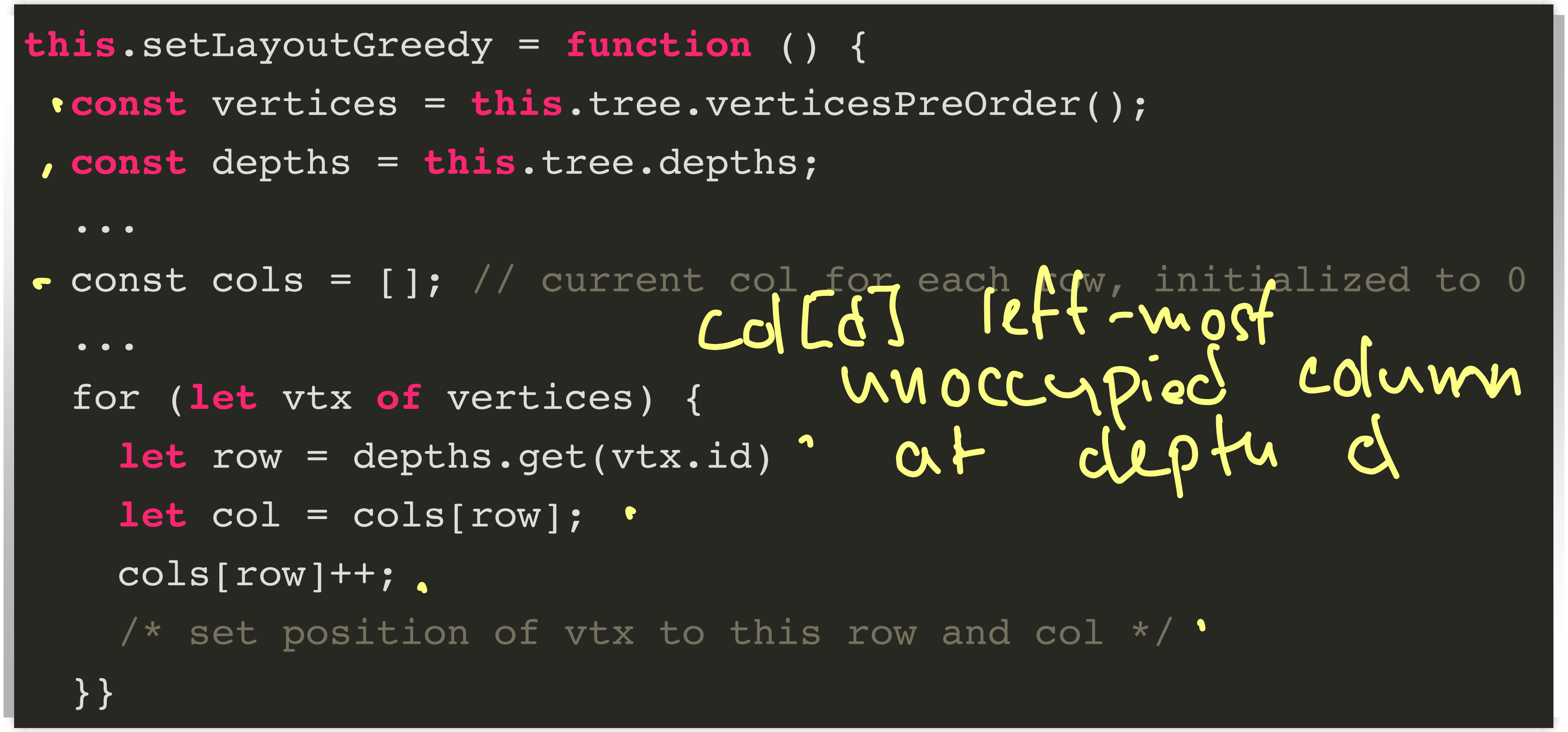
```
this.verticesPreOrder = function (from = this.root) {
  let vertices = [];
  vertices.push(from);
  if (from.left != null)
    vertices = vertices.concat(this.verticesPreOrder(from.left));
  if (from.right != null)
    vertices = vertices.concat(this.verticesPreOrder(from.right))
  return vertices;
```

left descendants

right descendants

Greedy Layout in JavaScript

Getting Rows and Columns



```
this.setLayoutGreedy = function () {
  const vertices = this.tree.verticesPreOrder();
  const depths = this.tree.depths;
  ...
  const cols = []; // current col for each row, initialized to 0
  ...
  for (let vtx of vertices) {
    let row = depths.get(vtx.id)
    let col = cols[row];
    cols[row]++;
    /* set position of vtx to this row and col */
}}
```

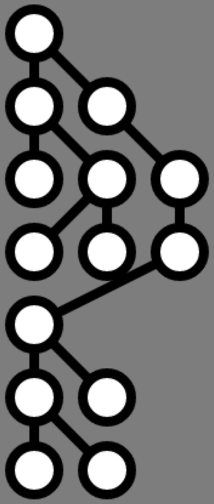
col[d] left-most unoccupied column at depth d

Demo

- `lec13-binary-tree.zip`

What is Missing?

lose visual rep of left/ right child



Second Principle

Aesthetic Principle 2. The left child of any node should appear to the left of its parent, and a right child should appear to the right of its parent.

How to Achieve Principles 1 and 2?

arrange columns so that "left-most" vertex is in first col, ...

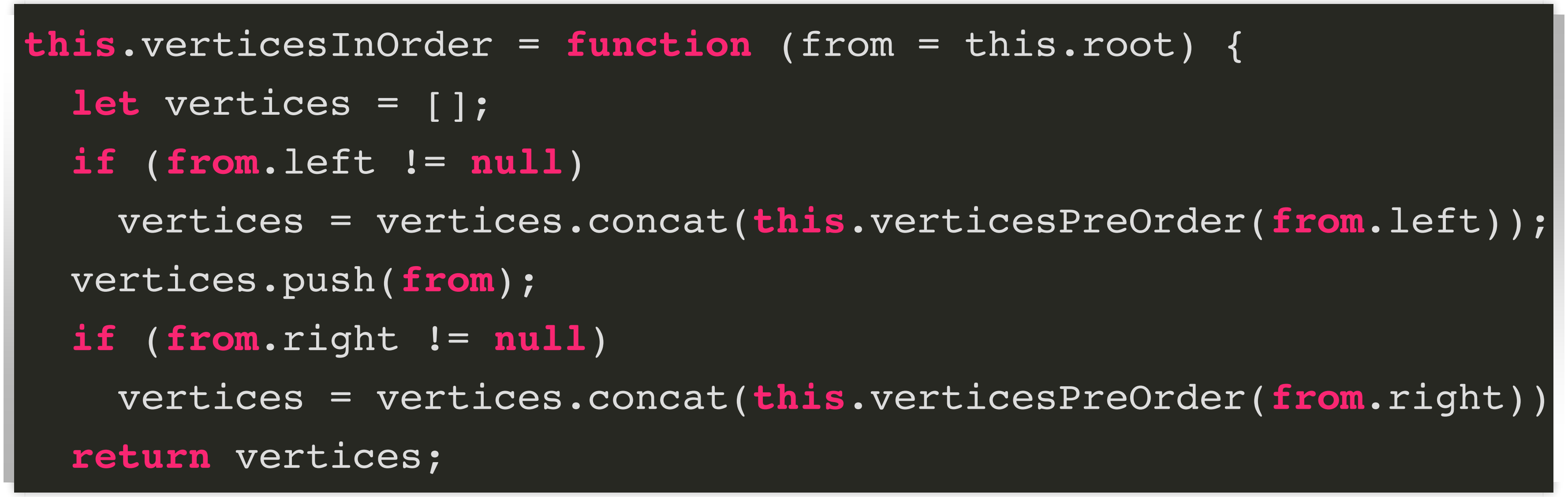
Knuth's Layout Algorithm

Rows and Columns

- rows are defined by depth (Aesthetic Principle 1)
- columns are "in-order" traversal order
    - each vertex gets own column
- guarantees
    - left descendants to the left
    - right descenadants to the right

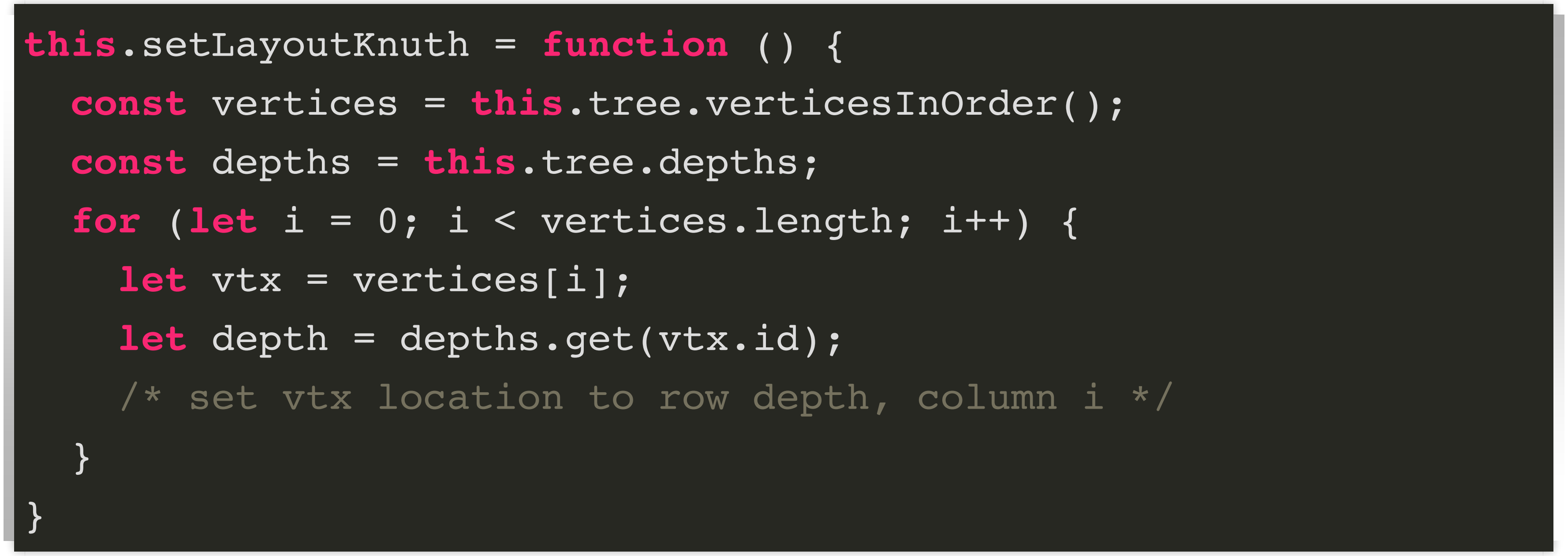
In-order Traversal

In-order Traversal in Code



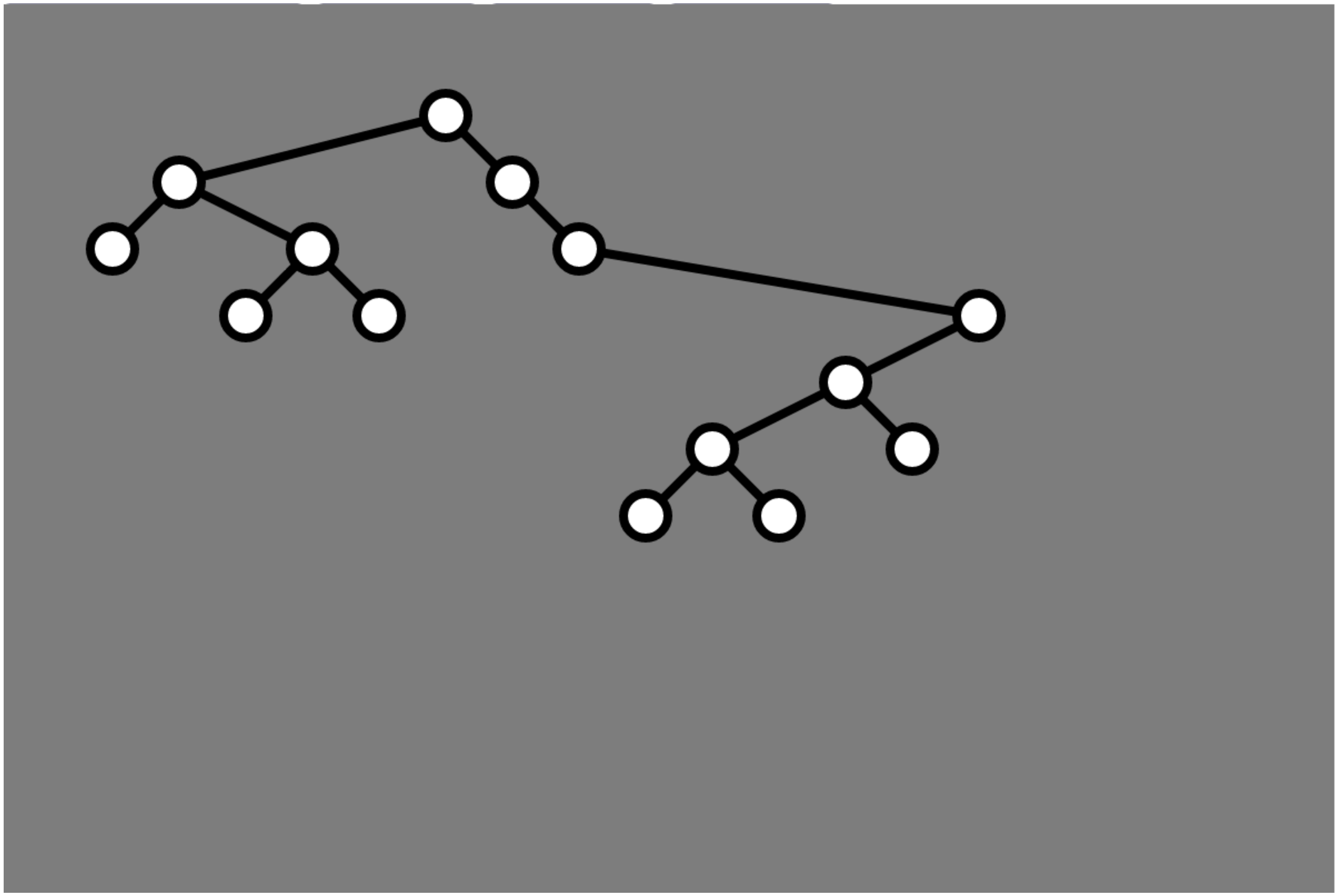
```
this.verticesInOrder = function (from = this.root) {
  let vertices = [];
  if (from.left != null)
    vertices = vertices.concat(this.verticesPreOrder(from.left));
  vertices.push(from);
  if (from.right != null)
    vertices = vertices.concat(this.verticesPreOrder(from.right))
  return vertices;
```

Knuth’s Layout in Code



```
this.setLayoutKnuth = function () {
  const vertices = this.tree.verticesInOrder();
  const depths = this.tree.depths;
  for (let i = 0; i < vertices.length; i++) {
    let vtx = vertices[i];
    let depth = depths.get(vtx.id);
    /* set vtx location to row depth, column i */
  }
}
```

Result

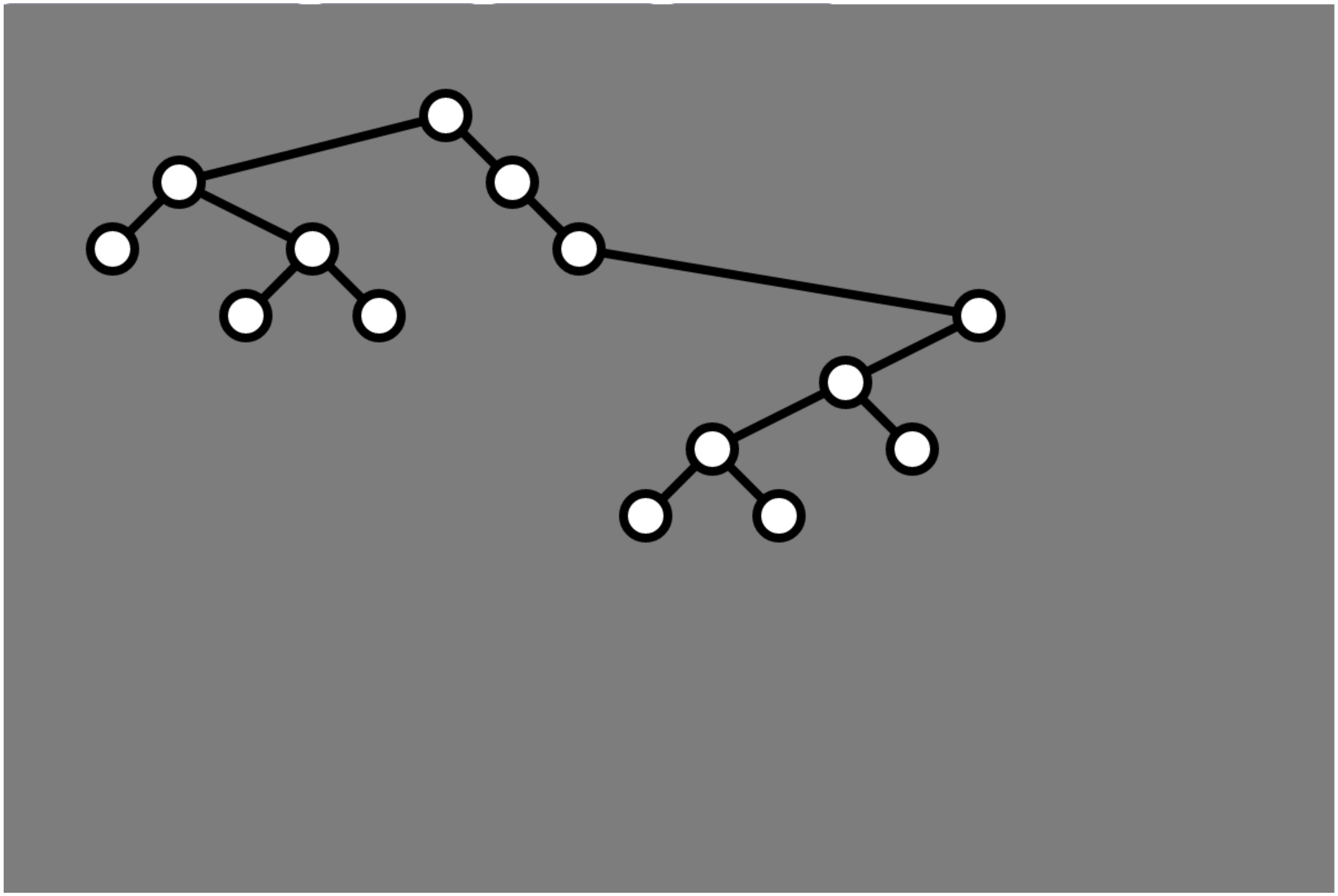


Demo, Again

- `lec13-binary-tree.zip`

What’s Not to Like?

Result Again



Third Principle

Aesthetic Principle 3. If a node has two children, it's x-coordinate should be the midpoint of its childrens' x-coordinates

Questions (Next Time)

1. How can we satisfy all three aesthetic principles?
2. How can all be satisfied while also minimizing the width of the drawing?
3. What tradeoffs are we forced to make balancinng these principles?