Lecture 12: Coordinate Transformations, Recursion & Self-similarity II

COSC 225: Algorithms and Visualization

Spring, 2023

Annoucements

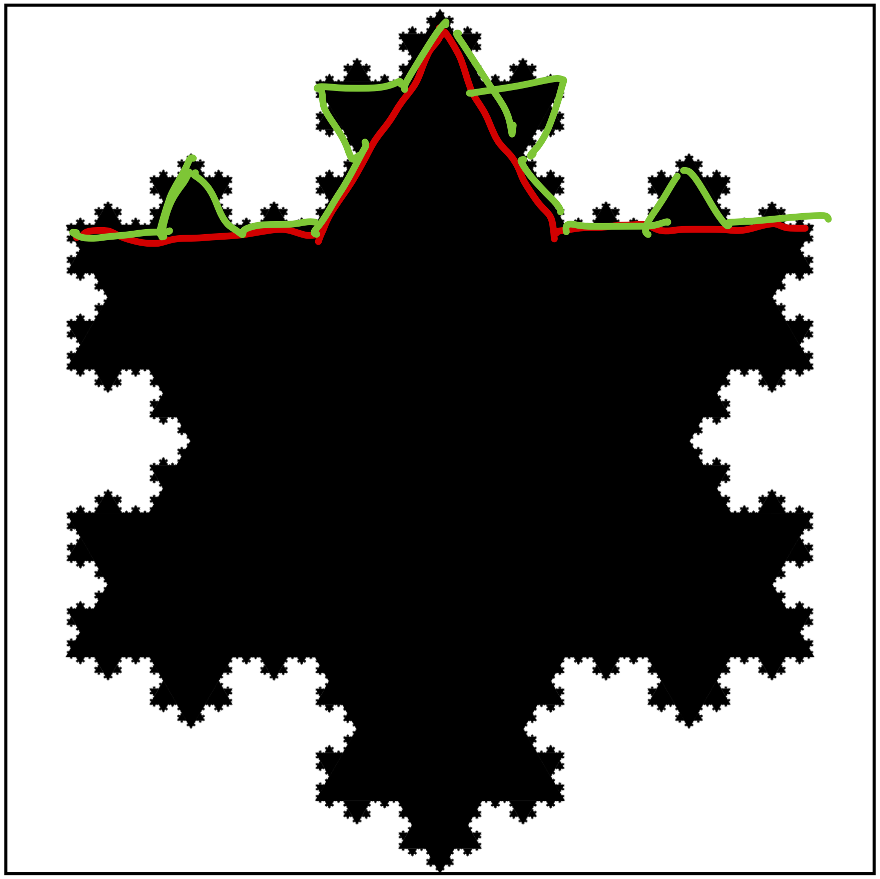
Assignment 06 Due ~~Friday~~ MONDAY!!!

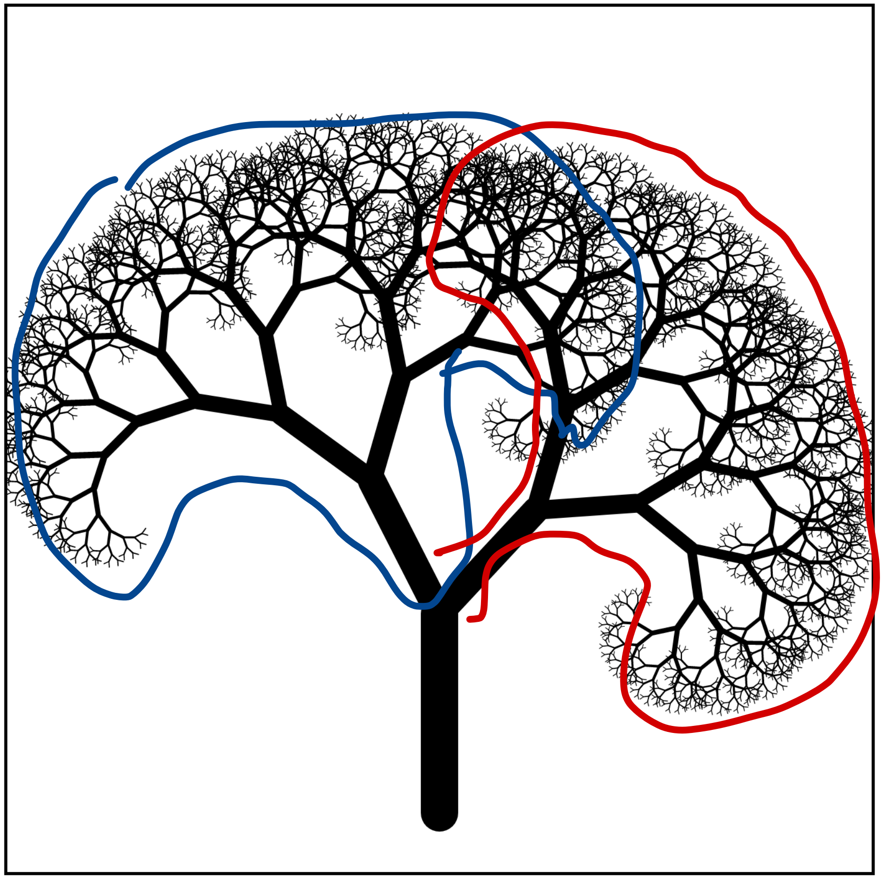
- tester later this week

Outline

1. Koch Curve
2. SVG Groups, Transformations, and Composition
3. Matrix Transformation Activity
4. Tree Example

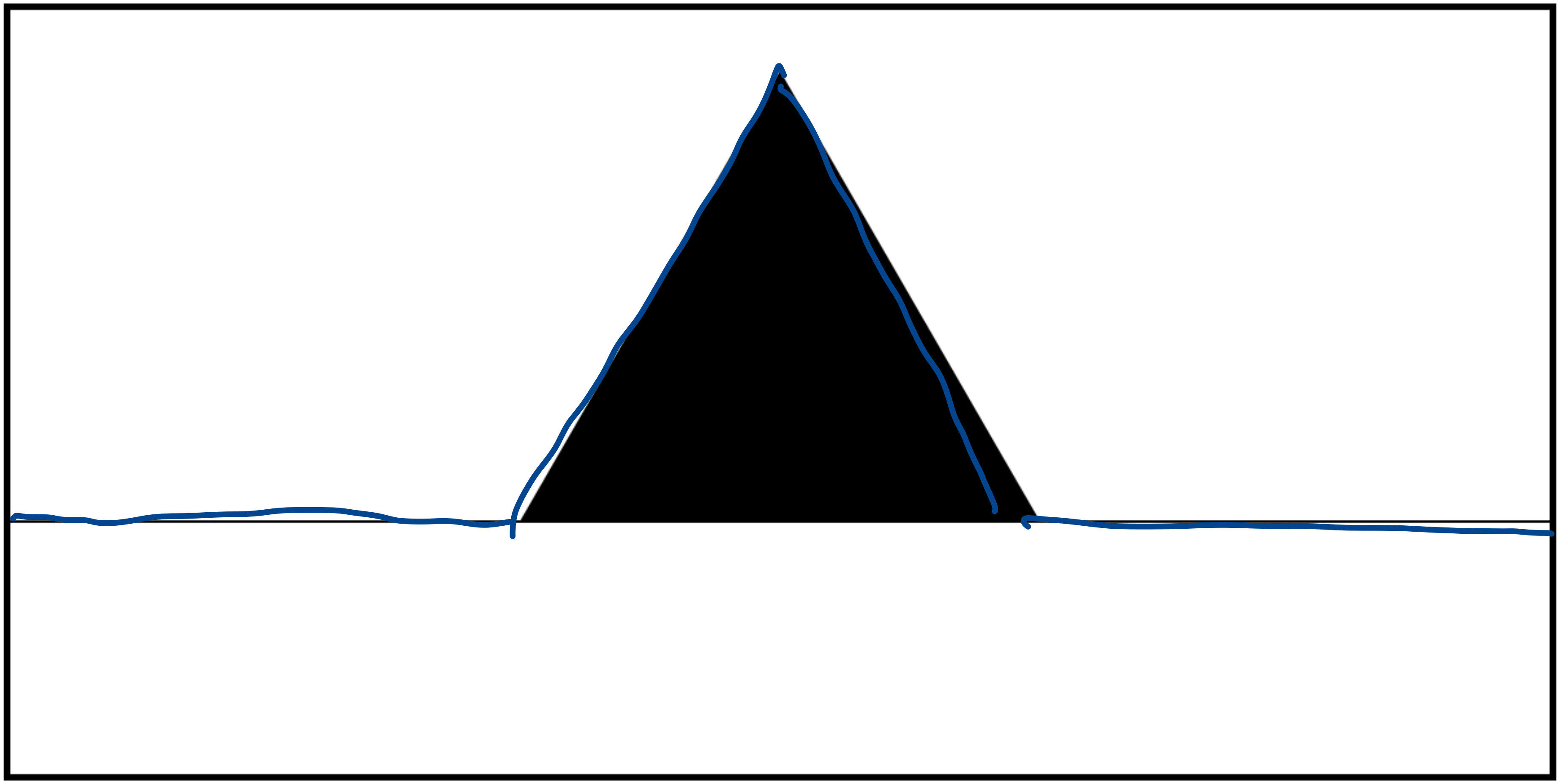
Motivation: Self-Similarity





Example: Koch Curve I

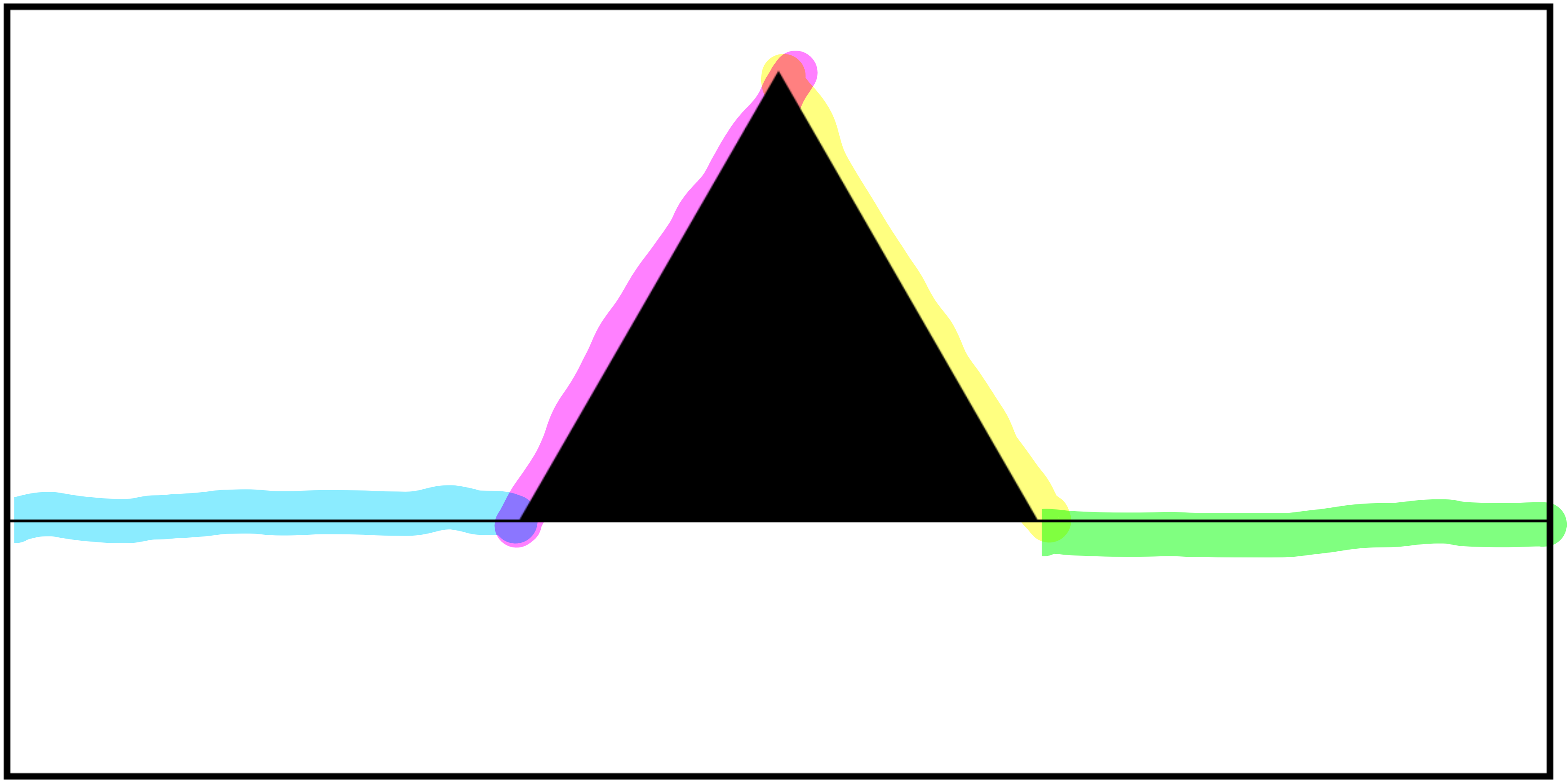
How did we make the snowflake fractal?



Step 1: define a basic shape

Example: Koch Curve II

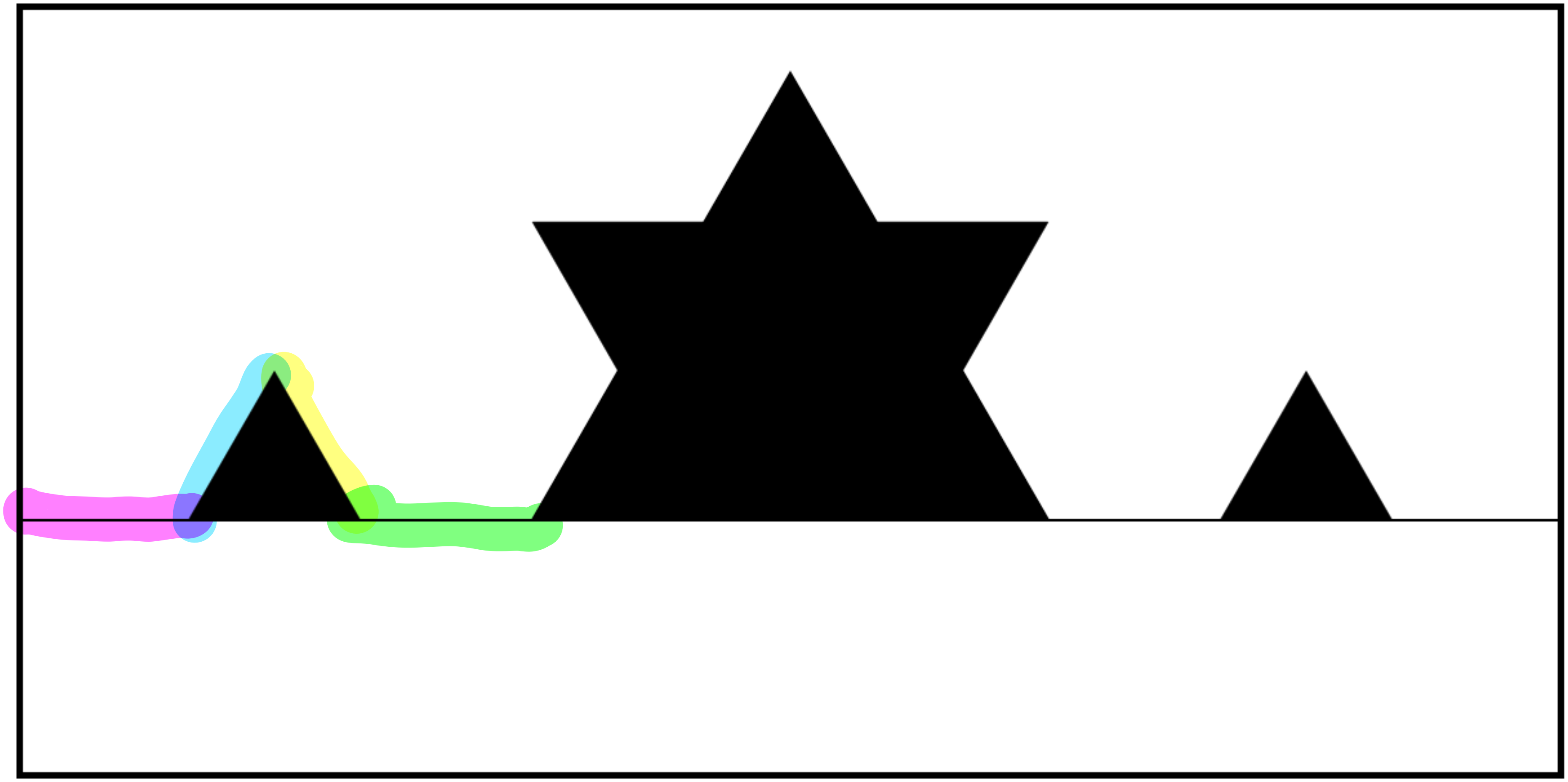
How did we make the snowflake fractal?



Step 2: define sub-shapes for basic shape

Example: Koch Curve III

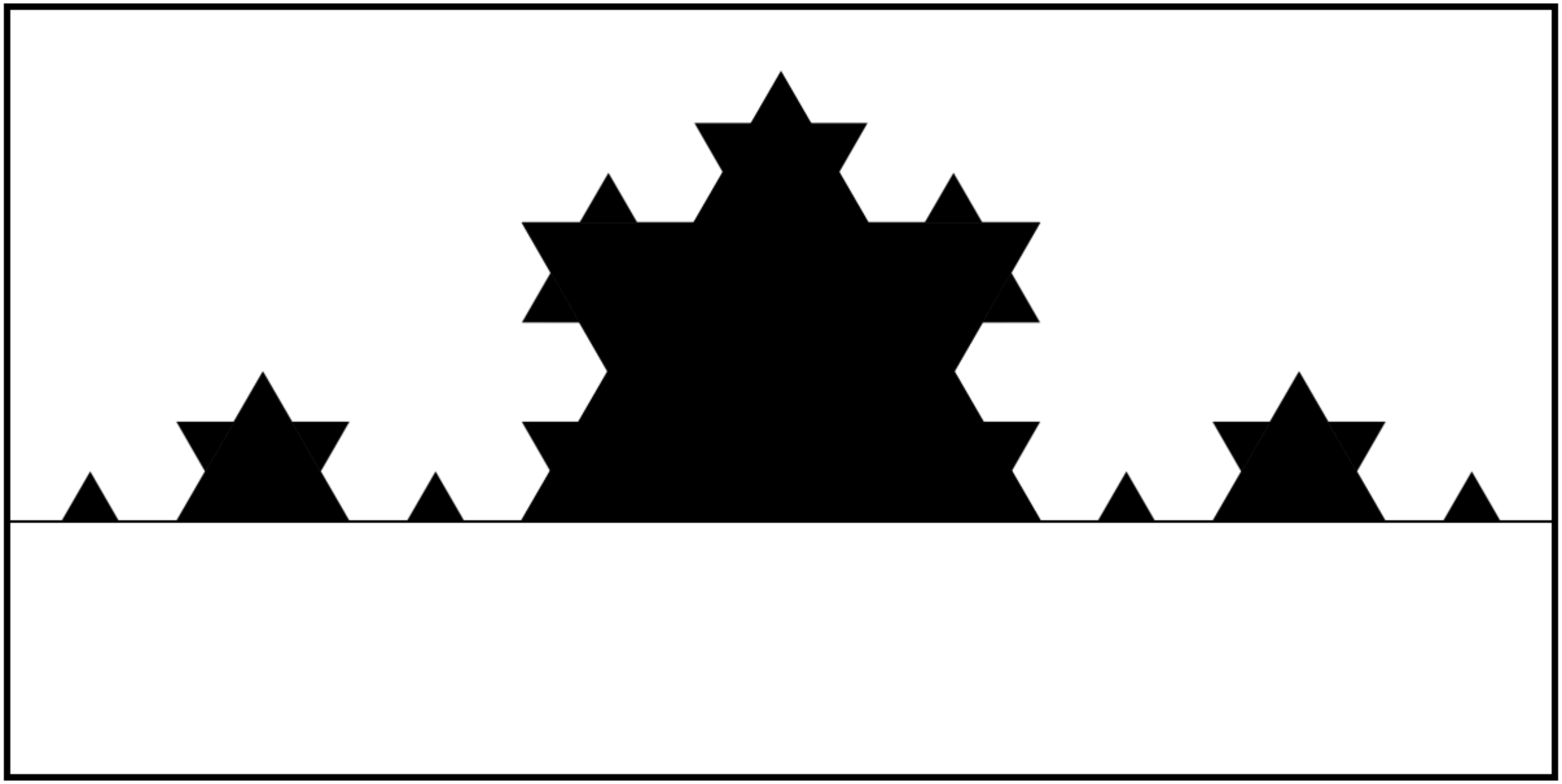
How did we make the snowflake fractal?



Step 3: recurse

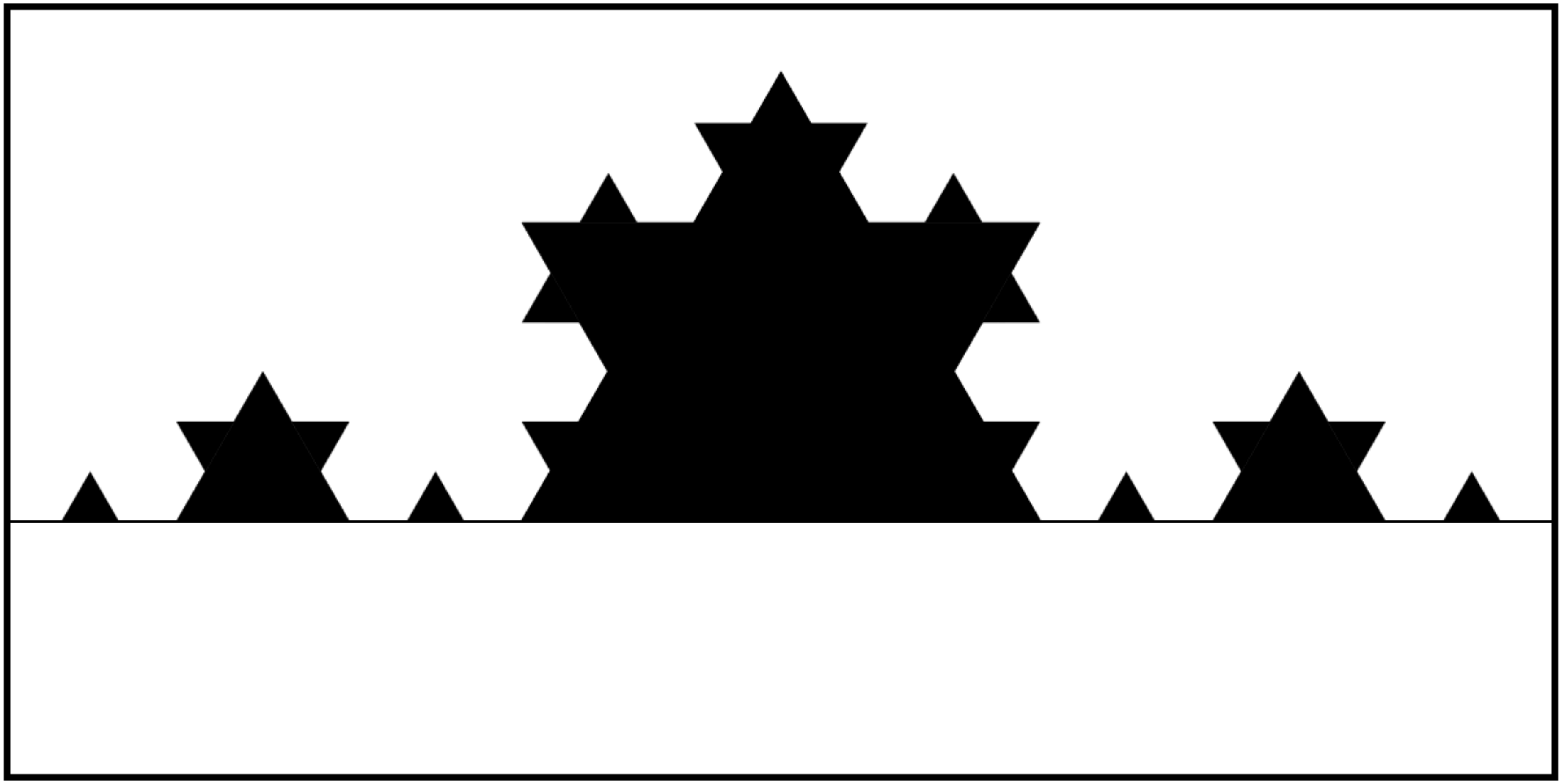
Example: Koch Curve IV

How did we make the snowflake fractal?



Step 3: recurse

Observation



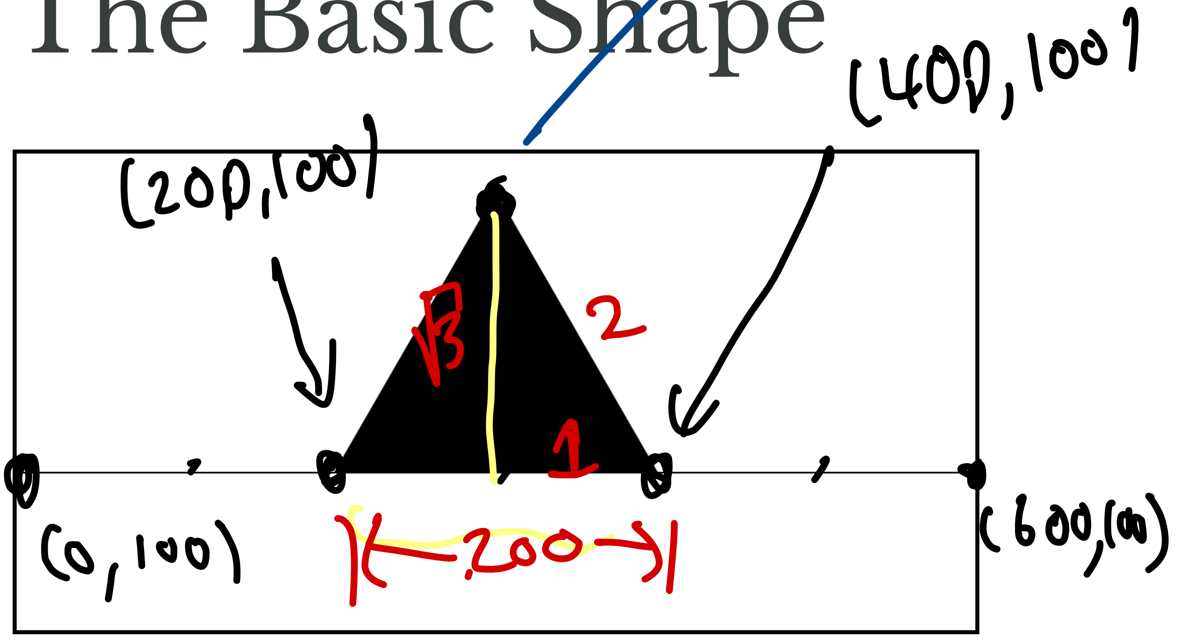
Each iteration draws a bunch of *transformed* copies of the original shape

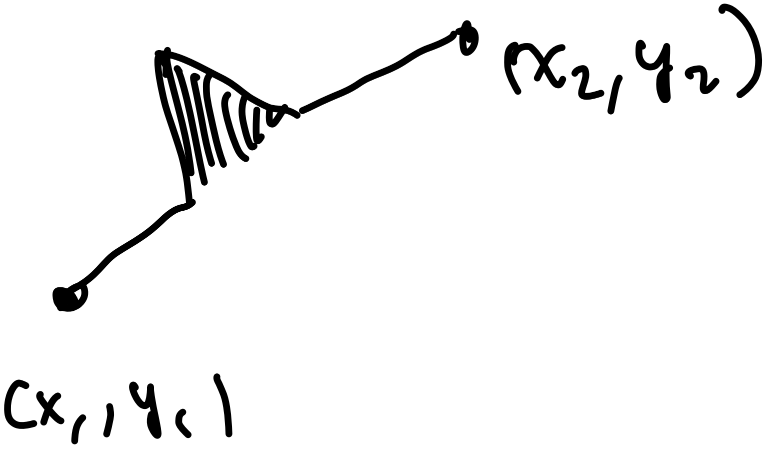
Activity

Draw two iterations of the Koch curve!

- `lec11-koch-step.zip`

The Basic Shape

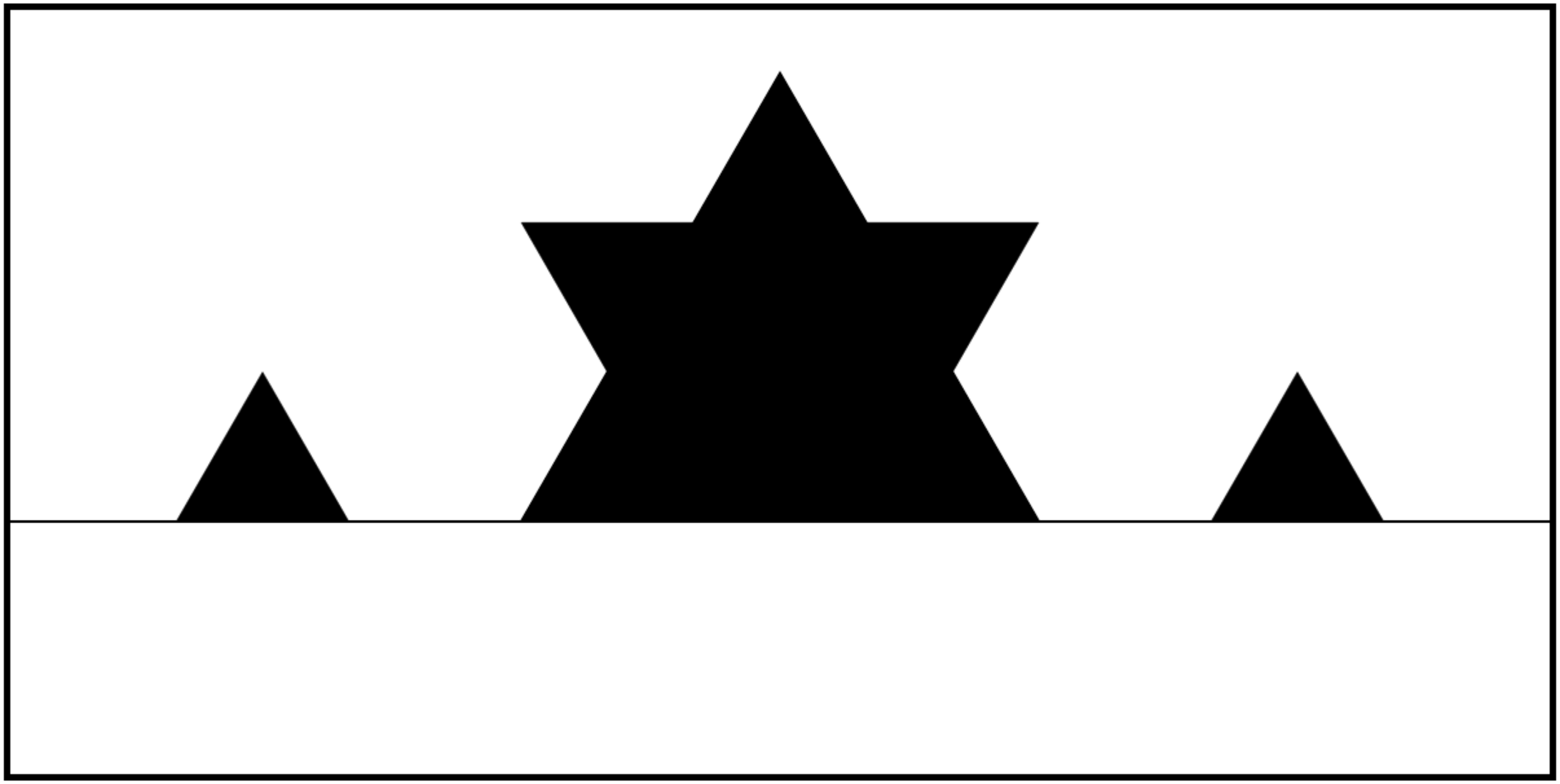




In `koch.js`:

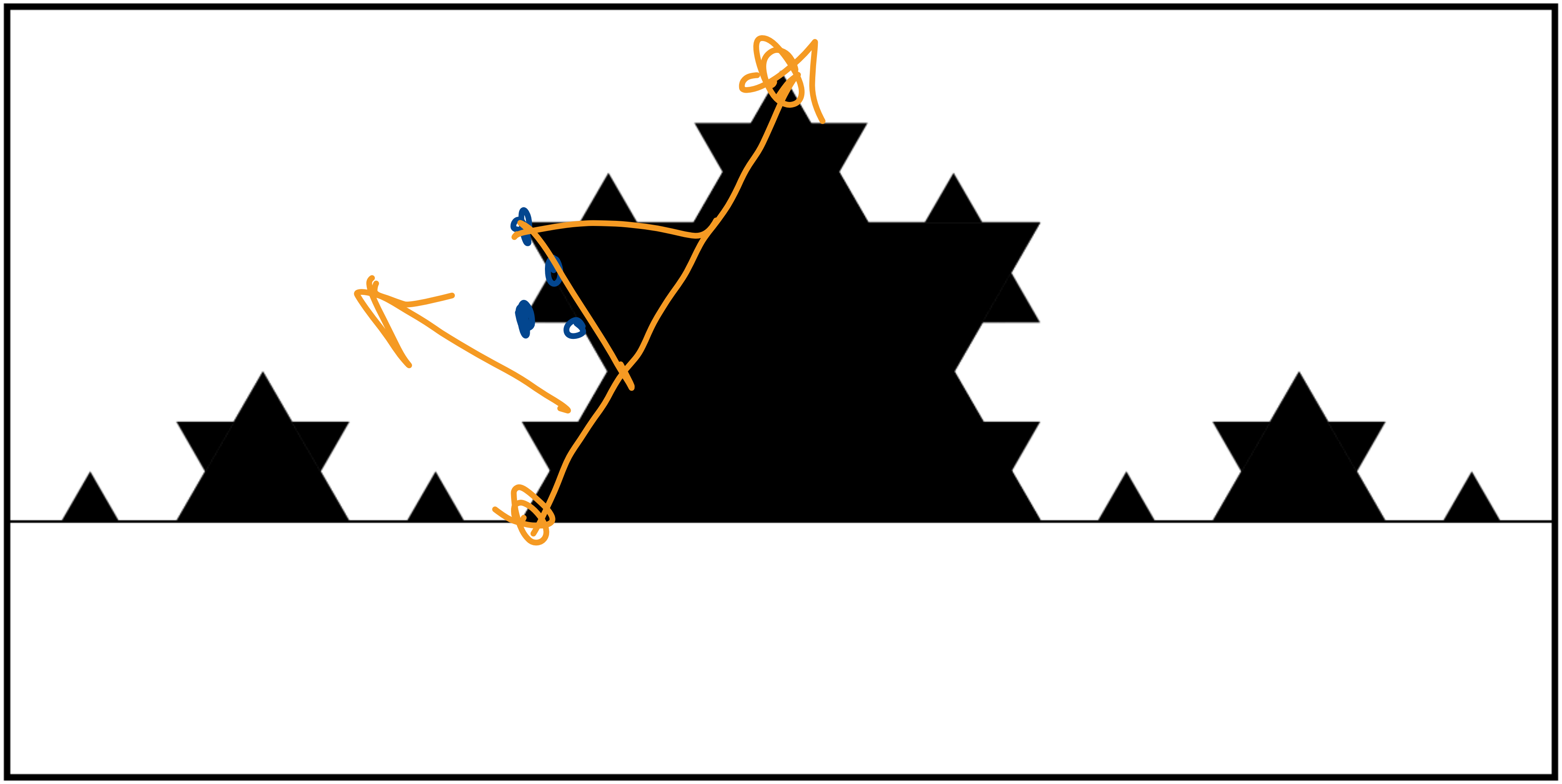
- `drawSegment(x1, y1, x2, y2)` will draw the the basic shape transformed start at `(x1, y1)` and end at `(x2, y2)`
- in original, `(x1, y1, x2, y2) = (0, 100, 600, 100)`,

How to Add First Iteration?

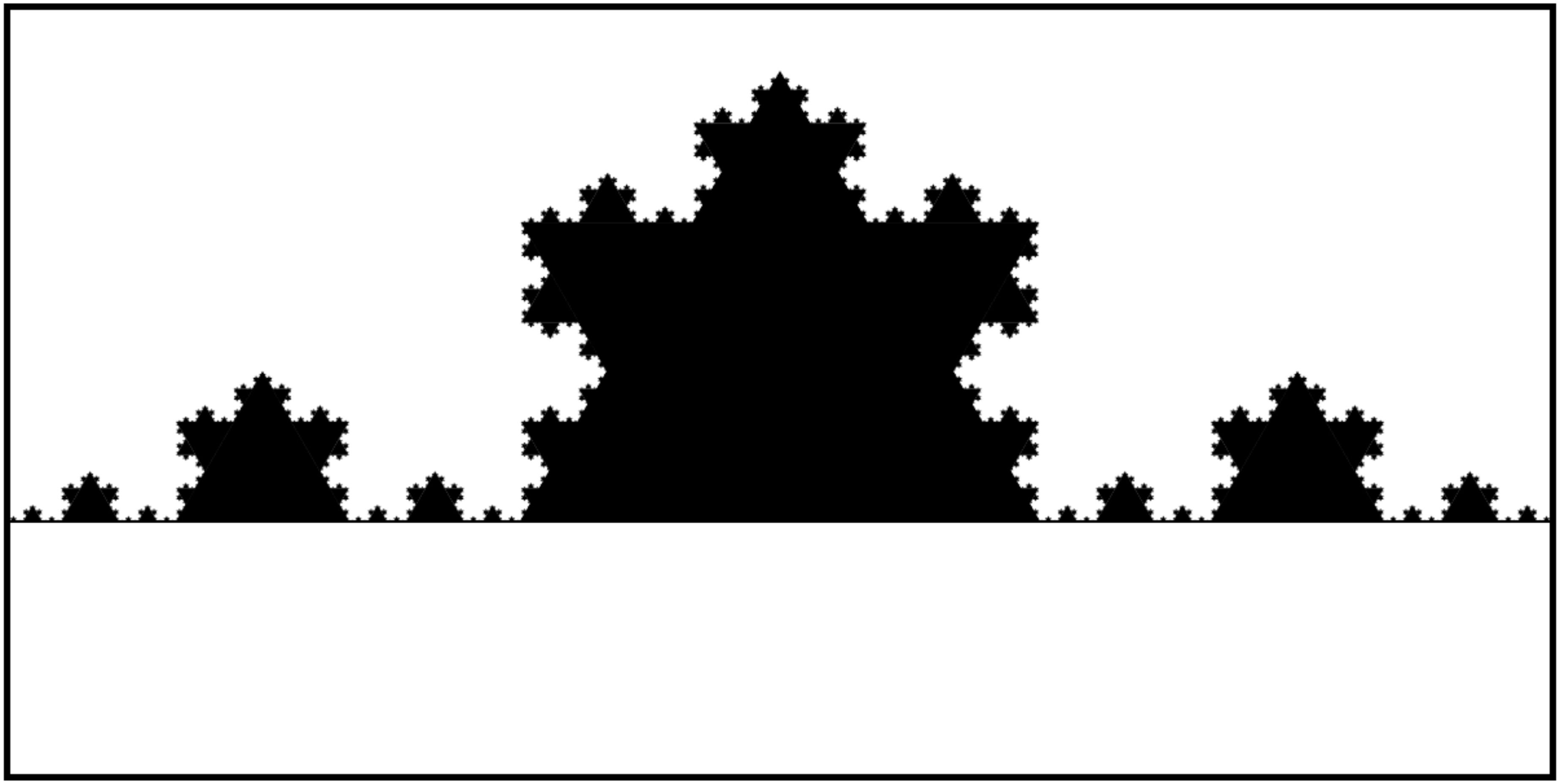


Find coordinates of endpts of sub-segments, + draw.

The Second Iteration?



This Would Be Annoying!

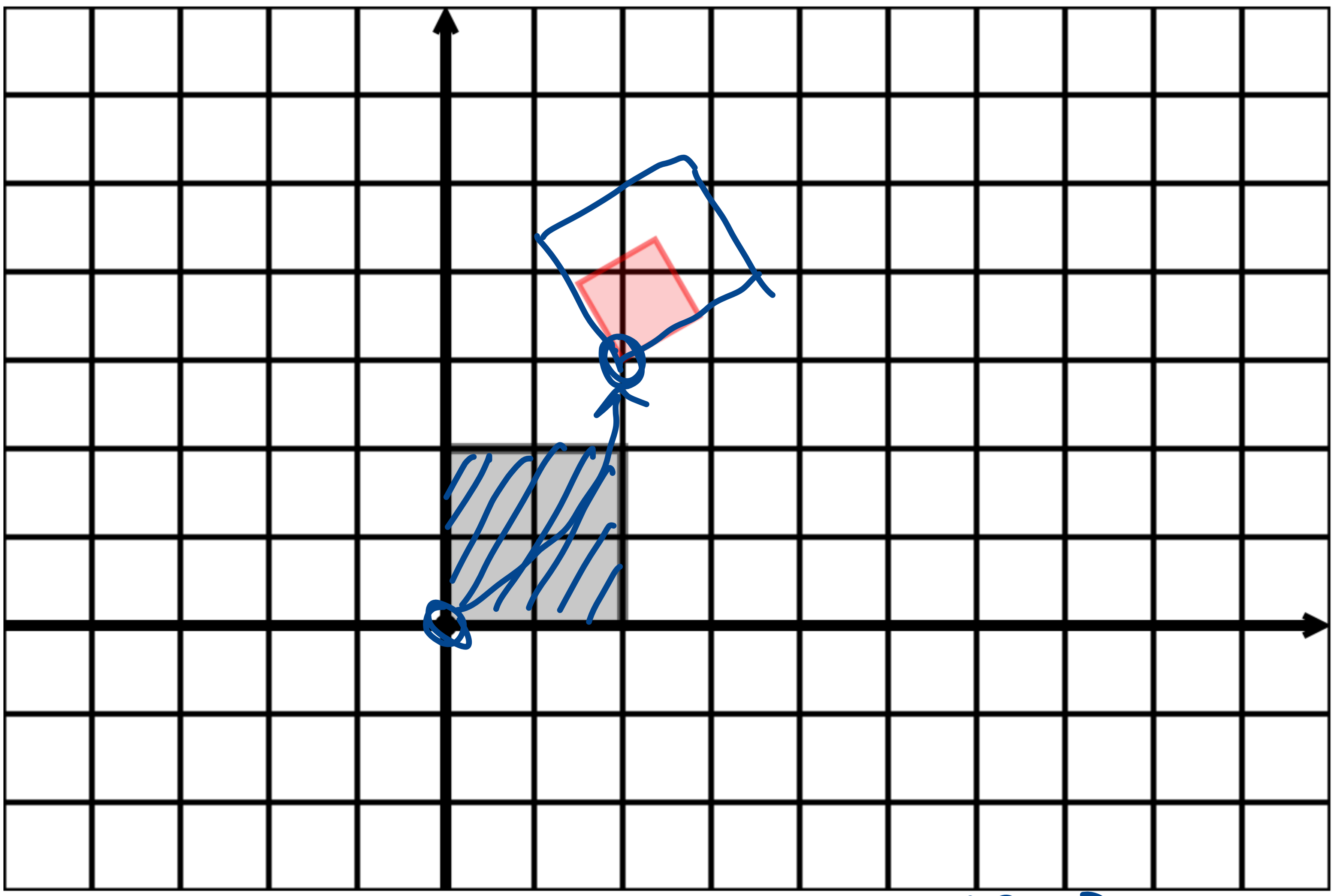


Composition

From last time: transformations **compose**

- perform transformation 1, then transformation 2
- transformation 2 is performed *relative* to transformation 1

Transformed

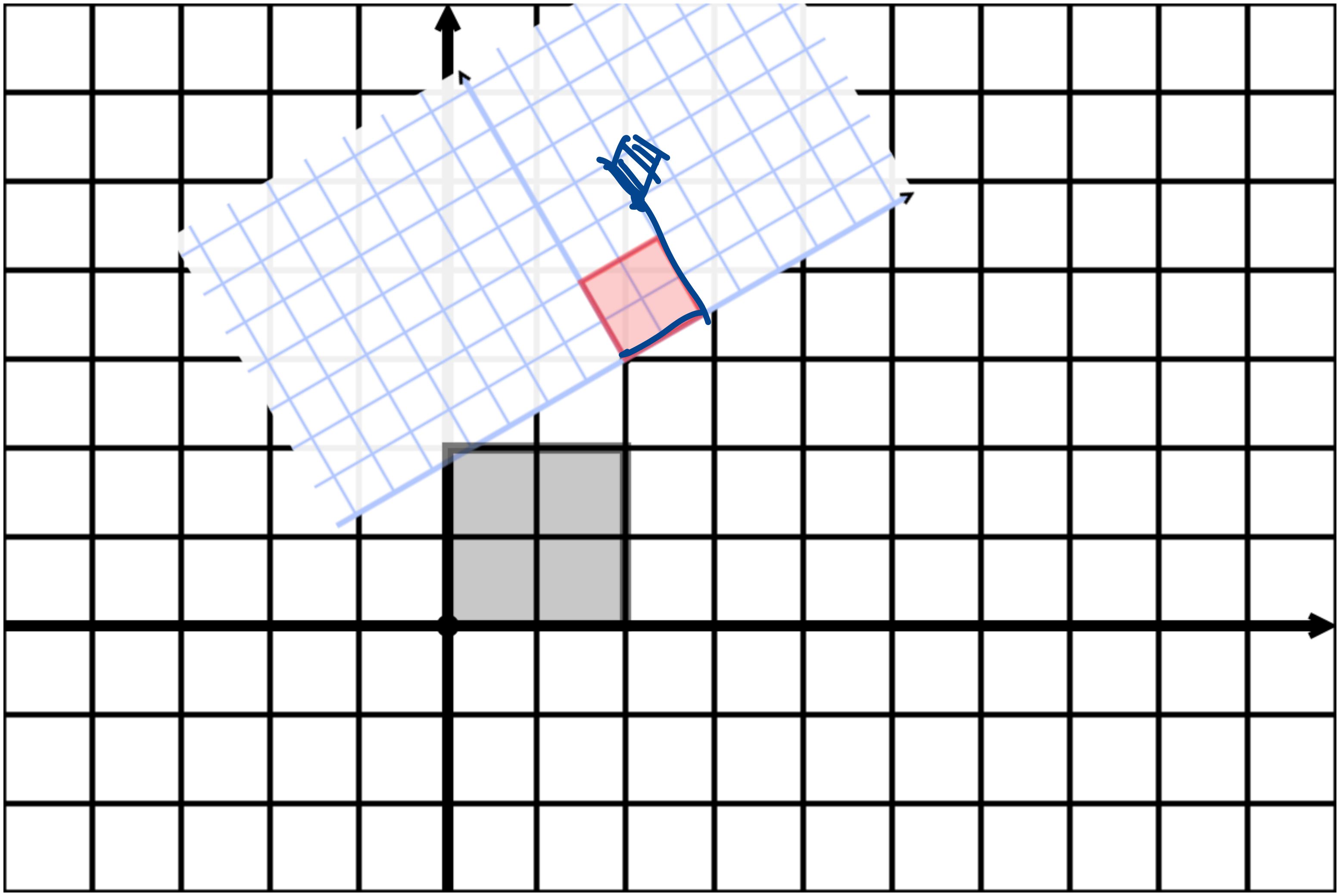


translate origin (2,3

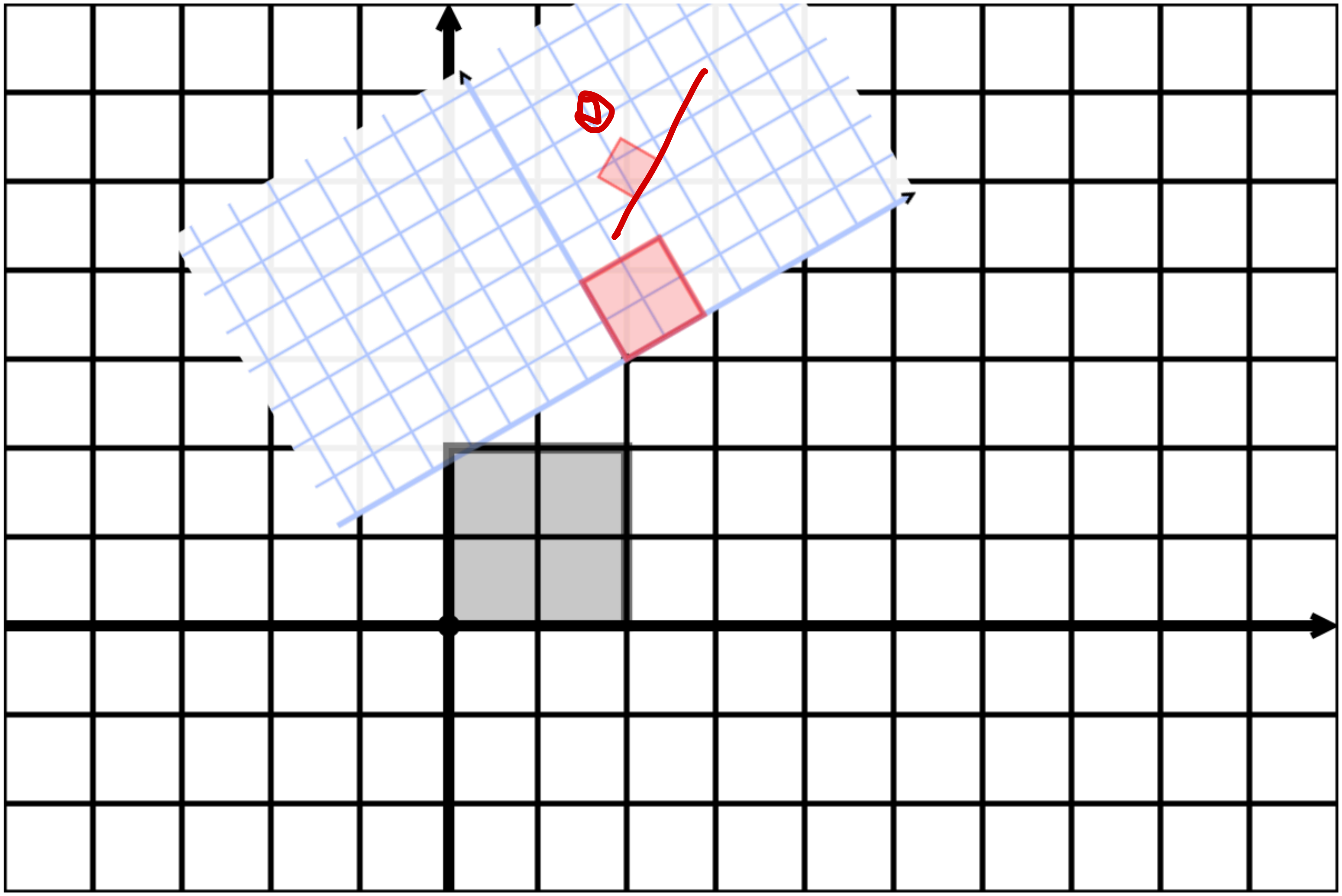
rotate 30°

scale (0.5)

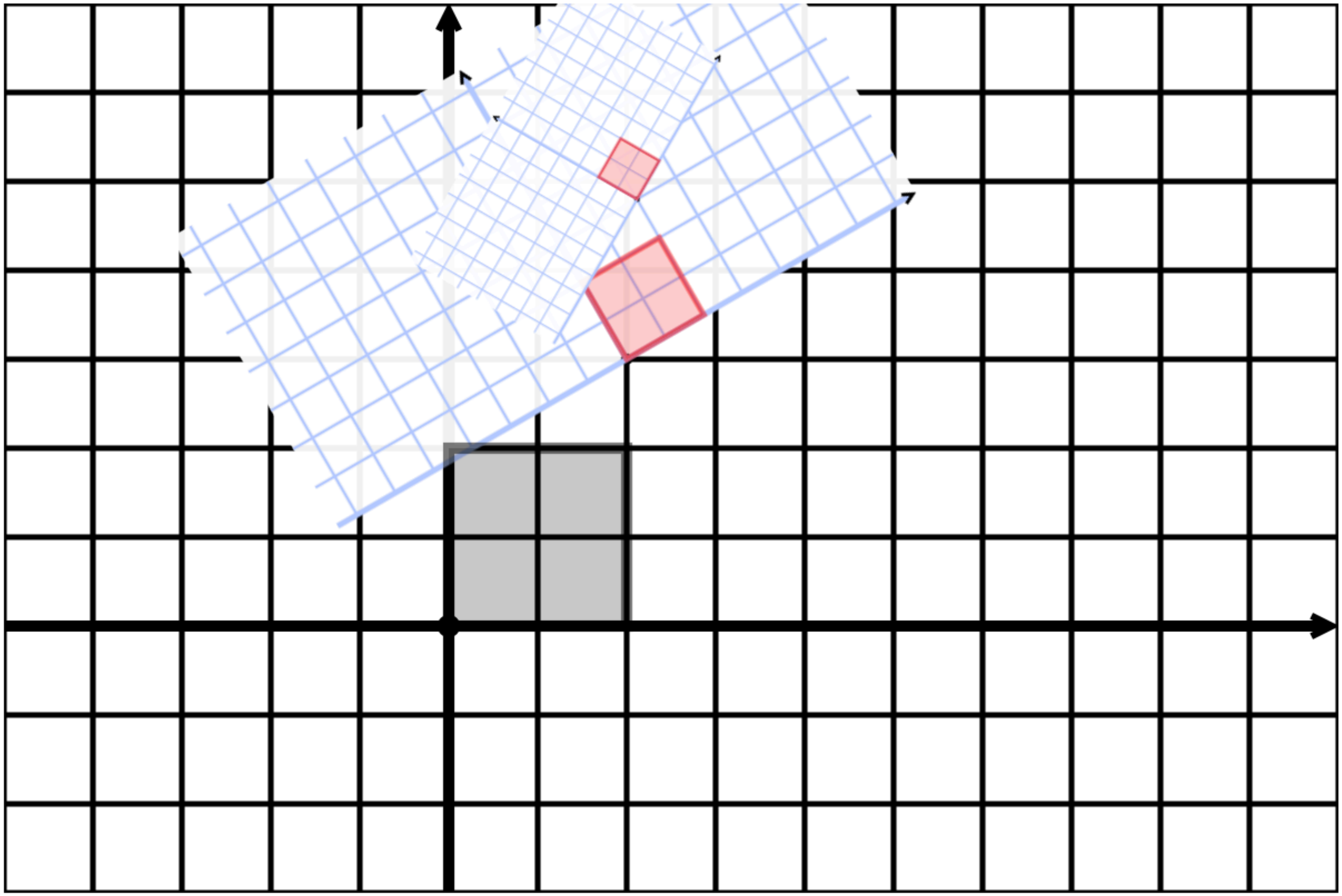
Transformed Coordinates



Element in New Coordinates



New New Coordinates

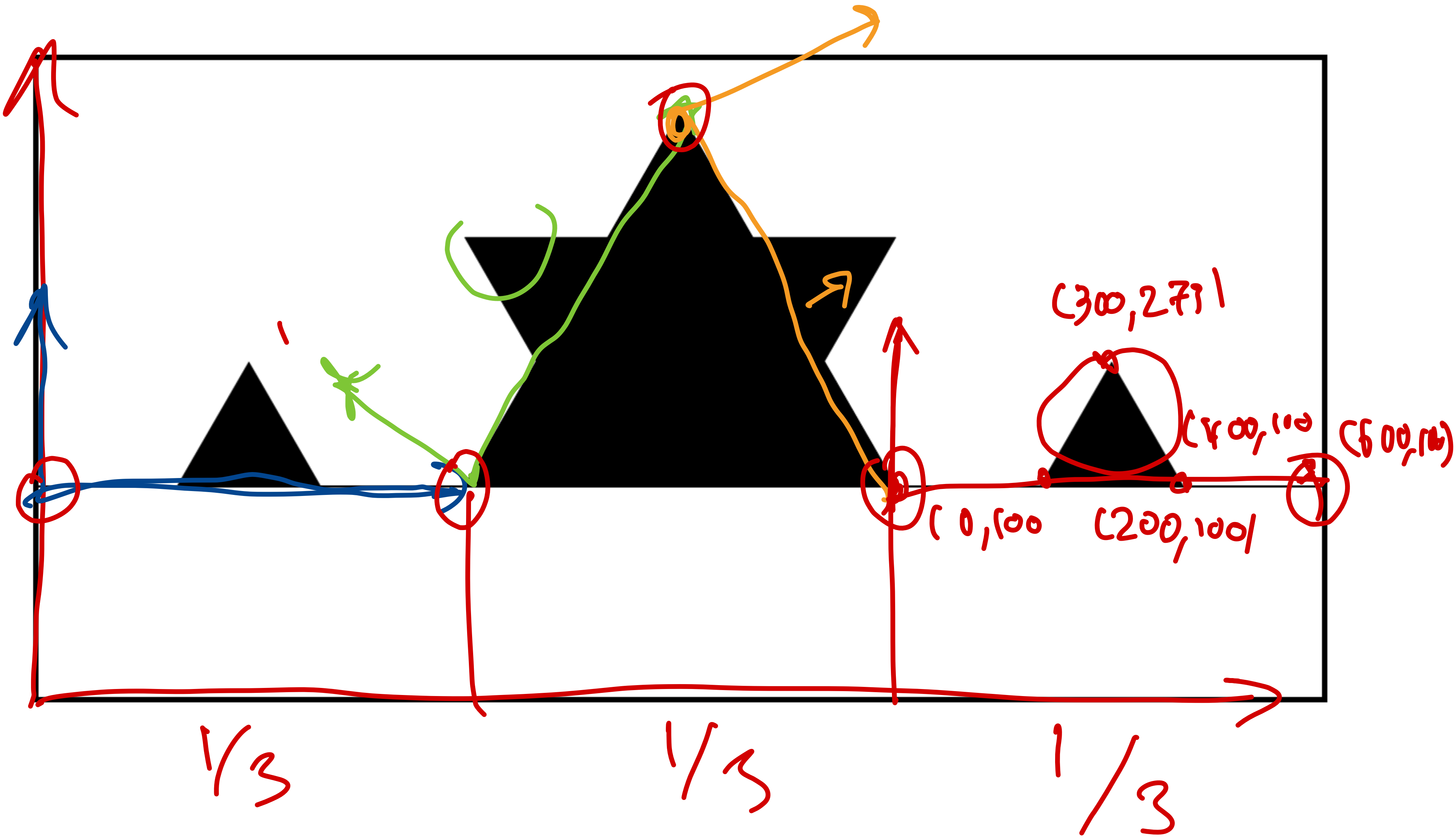


Koch Revisited

To draw a Koch segment:

1. Change to local coordinates for that segment
2. Draw a Koch segment
    - *relative to local coordinates* instructions are the same as original segment
3. Recursively draw a Koch segment on each sub-segment

Koch with Coordinates



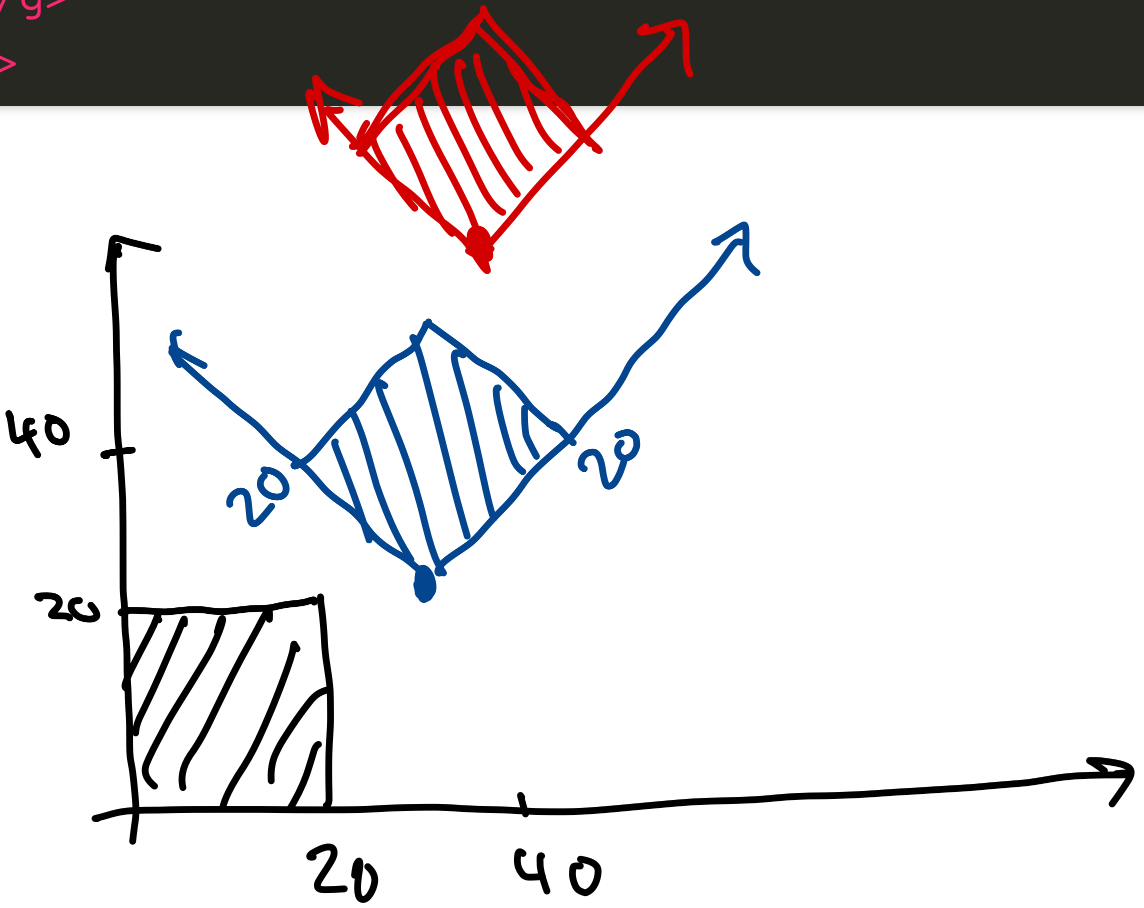
The <g> Element

In SVG, `<g>` is a **group** element

- all elements in the same `<g>` are drawn together
- transformations of `<g>` are applied to all elements in the `<g>`
- `<g>` elements can be nested
  - transformations of nested elements are *composed*

Example

```
<rect width="20" height="20" fill="black">
<g transform="translate(30, 20) rotate(45)">
  <rect width="20" height="20" fill="black">
  <g transform="translate(30, 20)">
    <rect width="20" height="20" fill="black">
  </g>
</g>
```



Orig coords

Drawing Koch Recursively

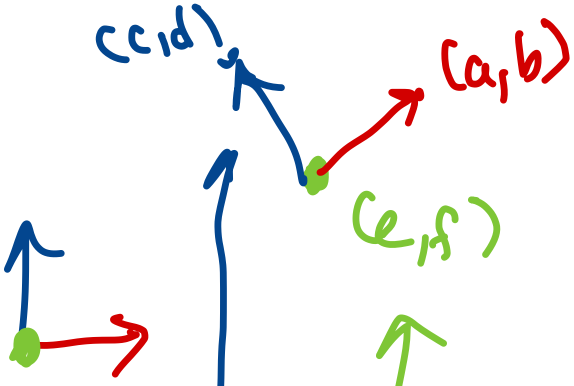
coords rel to Parent

```
DrawKoch(parentGroup, transformation):
    create and transform curGroup for this segment
    draw this segment to curGroup
    DrawKoch(curGroup, transformation for first sub-segment)
    DrawKoch(curGroup, transformation for second sub-segment)
    DrawKoch(curGroup, transformation for third sub-segment)
    DrawKoch(curGroup, transformation for fourth sub-segment)
```

Koch Demo

- `lec12-koch-step.zip`

The `matrix` Reloaded



In SVG you can perform an affine transformation

- vector $(1,0)$ to (a,b)
- vector $(0,1)$ to (c,d)
- point $(0,0)$ to (e,f)

with

```
transform=matrix(a, b, c, d, e, f)
```

(1,0)

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \sim \begin{pmatrix} s\cos\theta & -s\sin\theta \\ s\sin\theta & s\cos\theta \end{pmatrix}$$

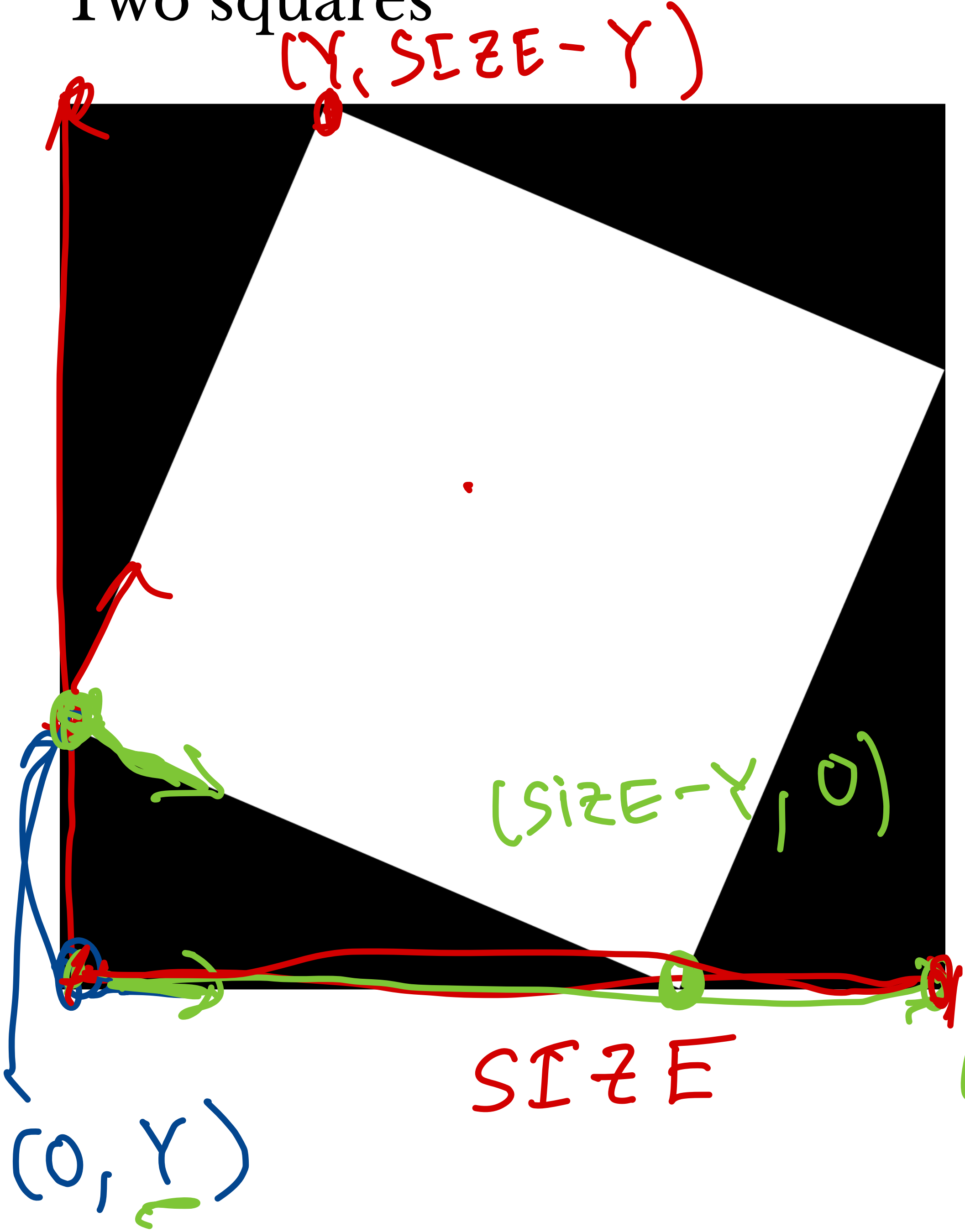
$$\begin{pmatrix} S & 0 \\ 0 & S \end{pmatrix}$$

scales by S

rotation by θ

For Your Consideration

Two squares



displacement:
(size-Y, -Y)

Matrix transformati

to transform outer square to inner

(0,0) ⟶ (SIZE,0)

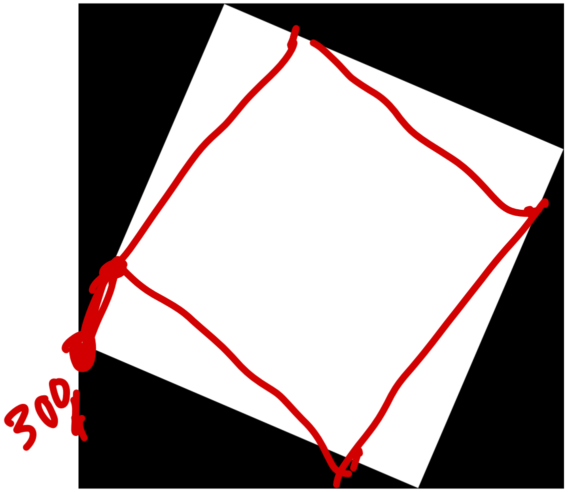
(0, Y)

↪ (size-Y, 0

$$\text{transform}(\underbrace{\boxed{\frac{\text{size}-Y}{\text{SIZE}}}\ \boxed{\frac{-Y}{\text{SIZE}}}}_{x}\ \underbrace{\boxed{\frac{Y}{\text{SIZE}}}\ \boxed{\frac{\text{SIZE}-Y}{\text{SIZE}}}\ 0\ Y}_{y})$$

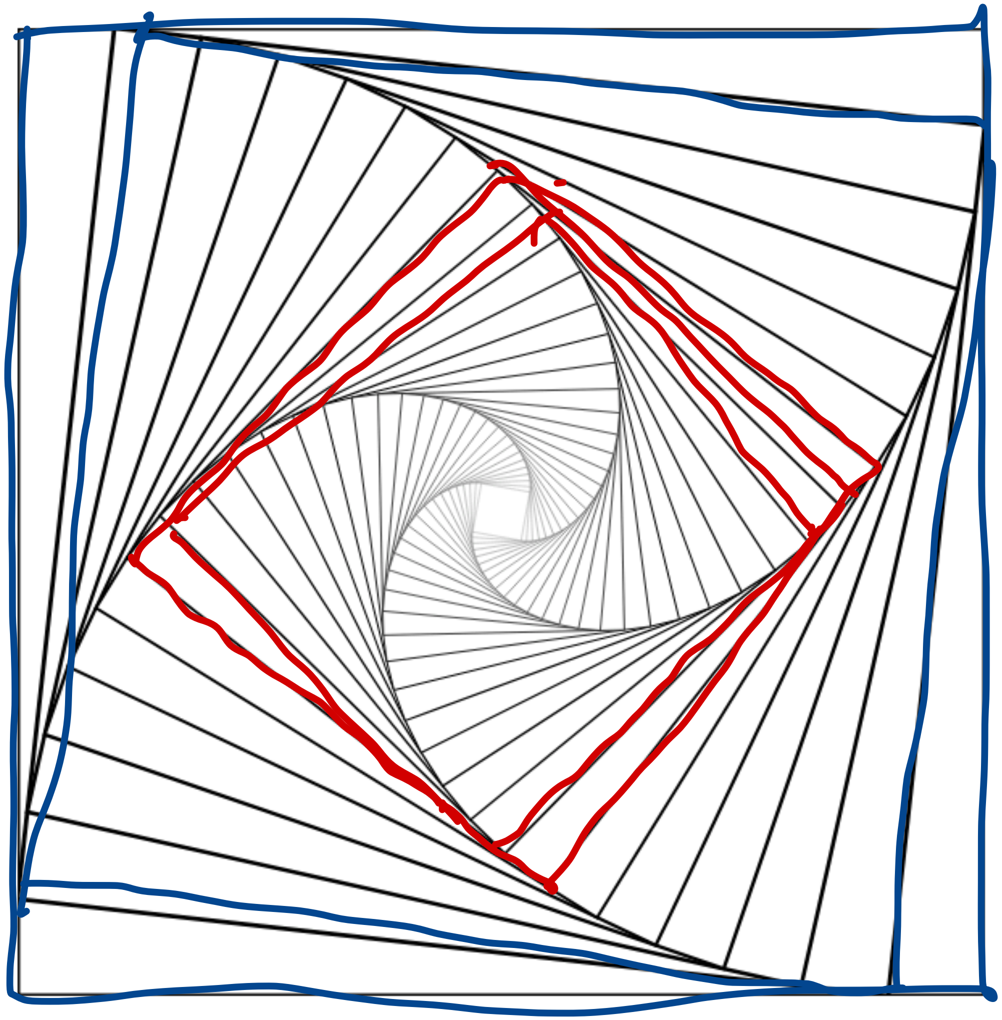
orig.

Activity: Find the `matrix`



Question. What `matrix` transformation will transform the outer square to the inner square?

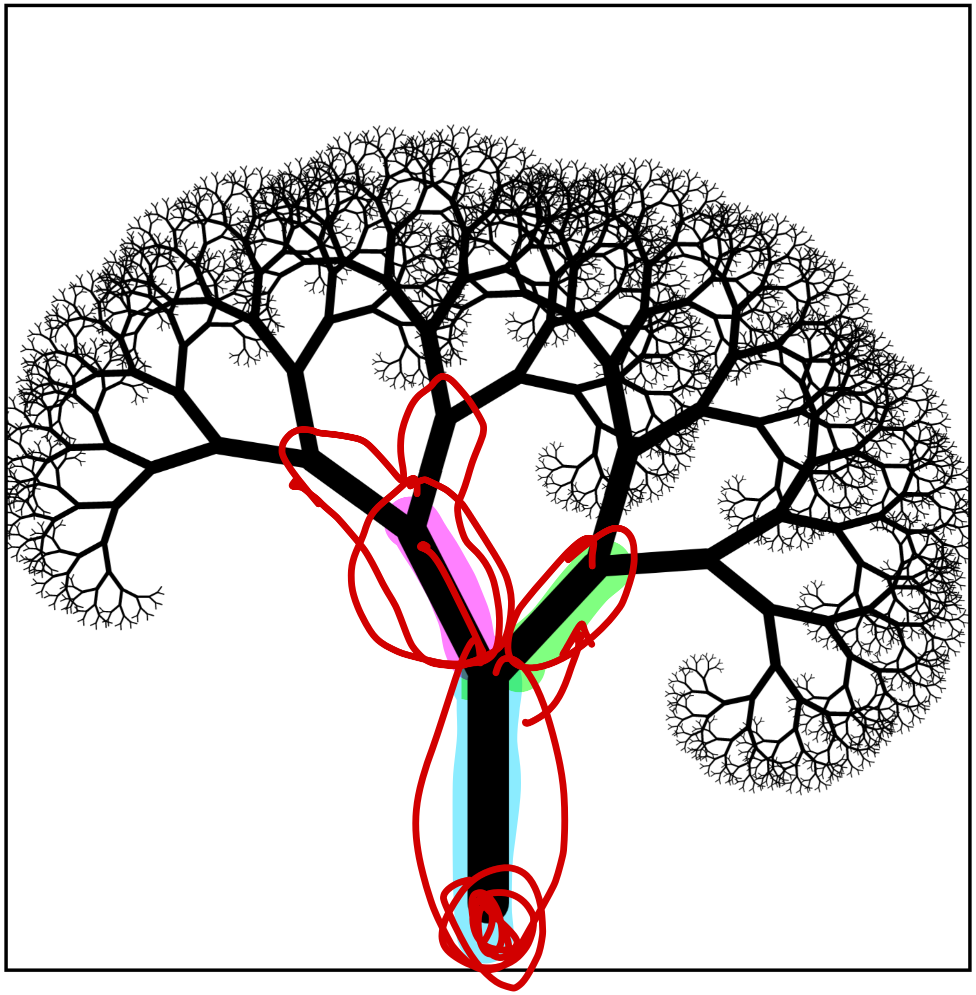
How Did I Make This?



Recursive Squares Demo

- `lec12-recusrive-squares.zip`

How Did I Make the Tree?



To Identify

1. Basic Shape

2. Recursive Step: transformation(s)?

Recursive Tree Demo

- `lec12-recursive-tree.zip`

Whoa, Dude

- `lec12-animated-tree.zip`

Assignment 07

Make a website that incorporates (recursive) self-similar graphics

- must change some attribute by recursion depth

Next Week

Automated Graph Drawing!

- Given (only) sets of vertices and edges of a graph, how can we draw it so that it looks nice?