Lecture 11: Coordinate Transformations, Recursion & Self-similarity I

COSC 225: Algorithms and Visualization

Spring, 2023

Annoucements

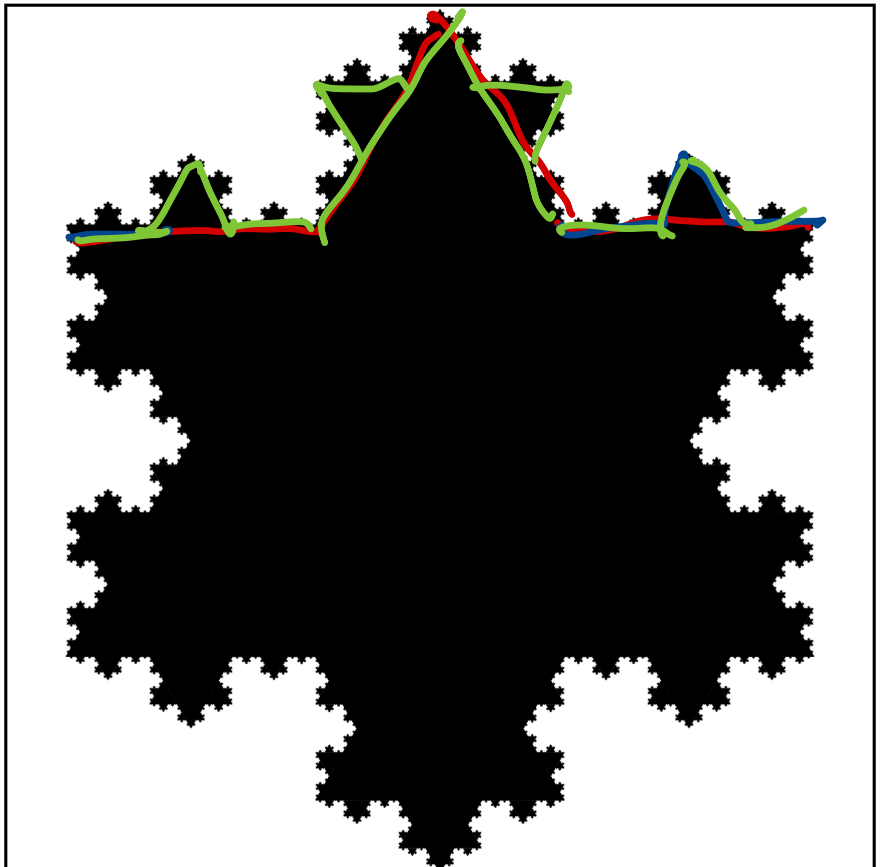
Assignment 06 Due ~~Friday~~ MONDAY!!!

- tester later this week

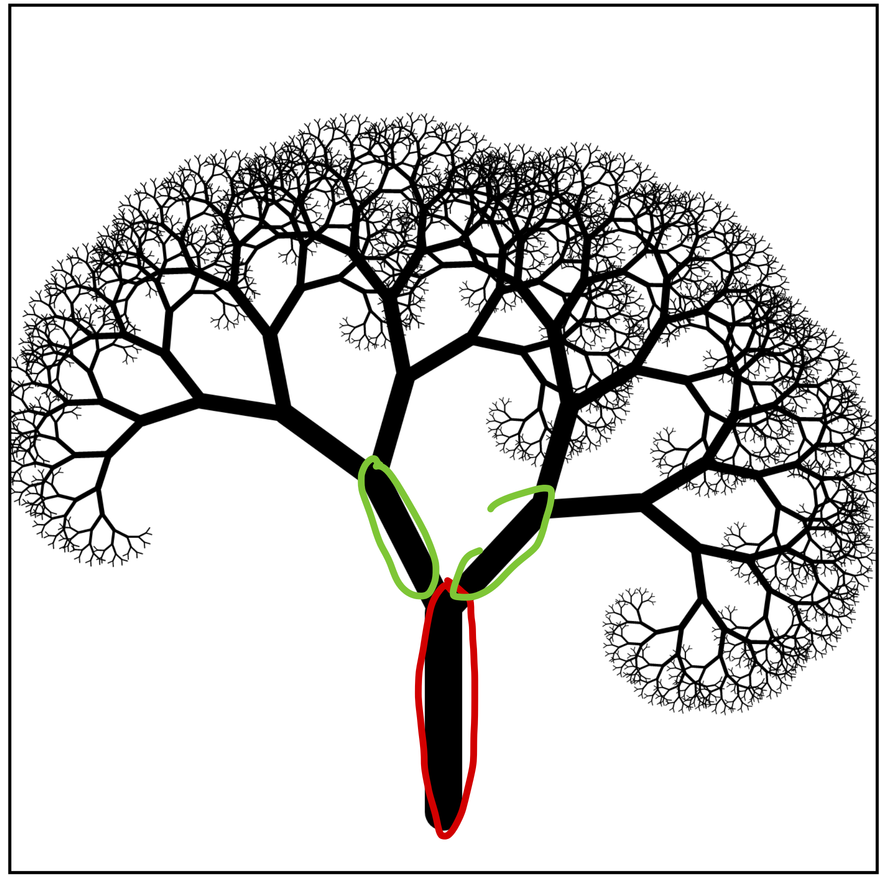
Outline

1. Coordinates
2. Coordinate Transformations
3. Koch Curve Activity
4. SVG Groups and Coordinates

Motivation: Self-Similarity



Koch Snowflake



Goal

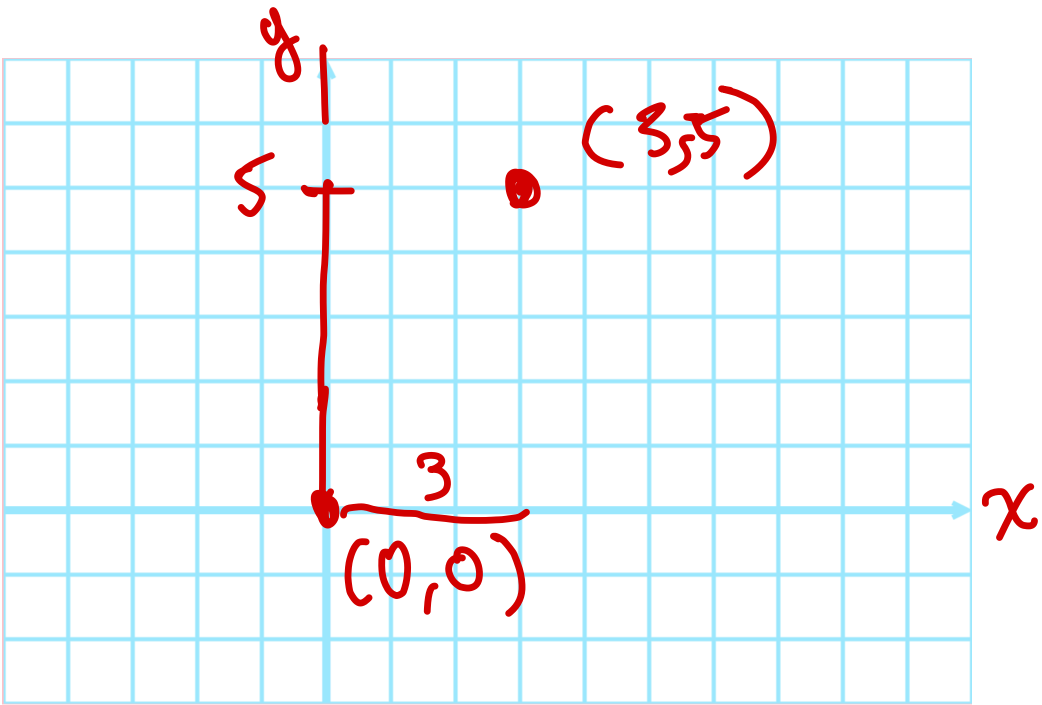
Generate self-similar graphical content

Homework 07: draw self-similar images

Coordinates

Cartesian Coordinates:

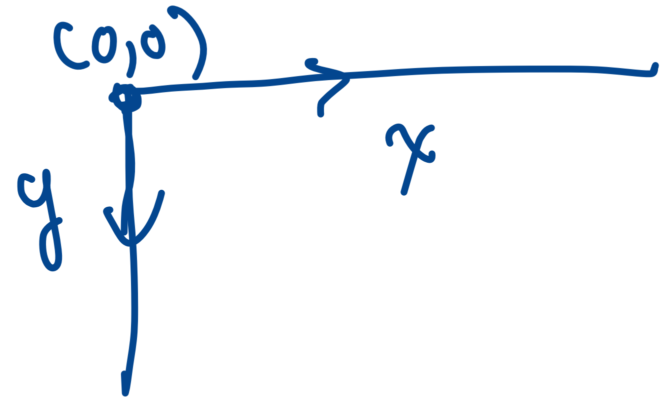
- associate each point in the plane with a pair of numbers: $A = (x, y)$



Screen vs Standard Coordinates

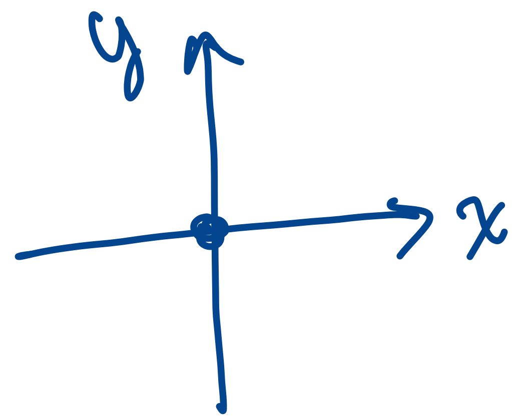
Screen coordinates:

- origin is upper left corner
- x increases in right direction
- y increases in *downward* direction

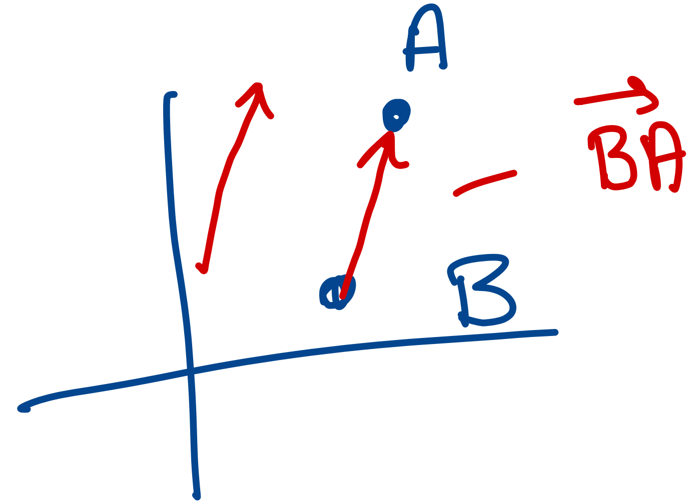


Standard coordinates:

- origin is somewhere
  - depends on region we want to depict
- x in right direction
- y increases in *upward* direction

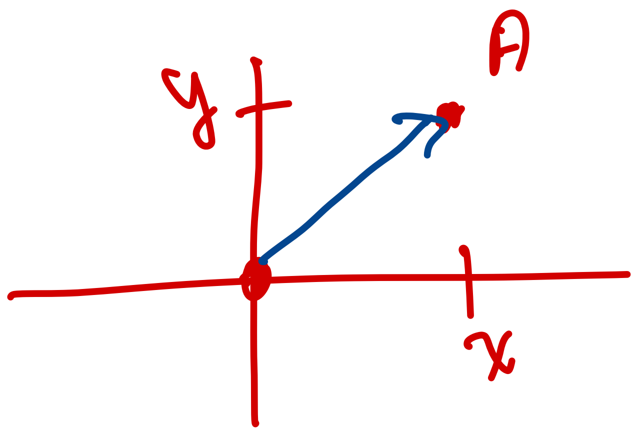


Today: use standard coordinates!

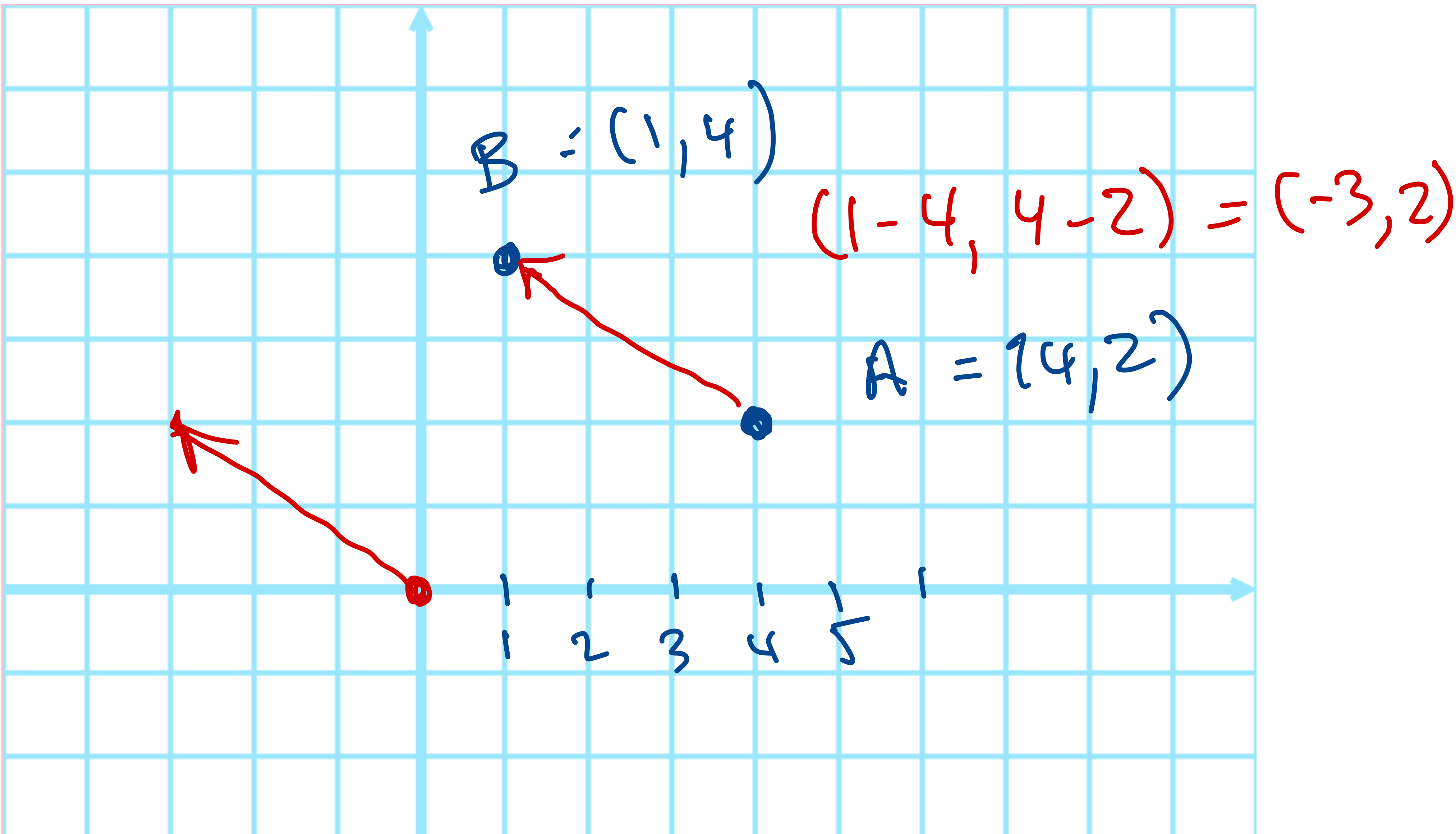


Points and Vectors

- a *point* is a *location* in the plane
  - specified by a pair of numbers: *coordinates*
- a *vectors* is a *displacement* between points
  - magnitude + direction
  - *arrows*
  - specified by a pair of numbers: *components*



Points and Vectors, Illustrated



Vector Operations

Vectors can be manipulated with algebraic operations:

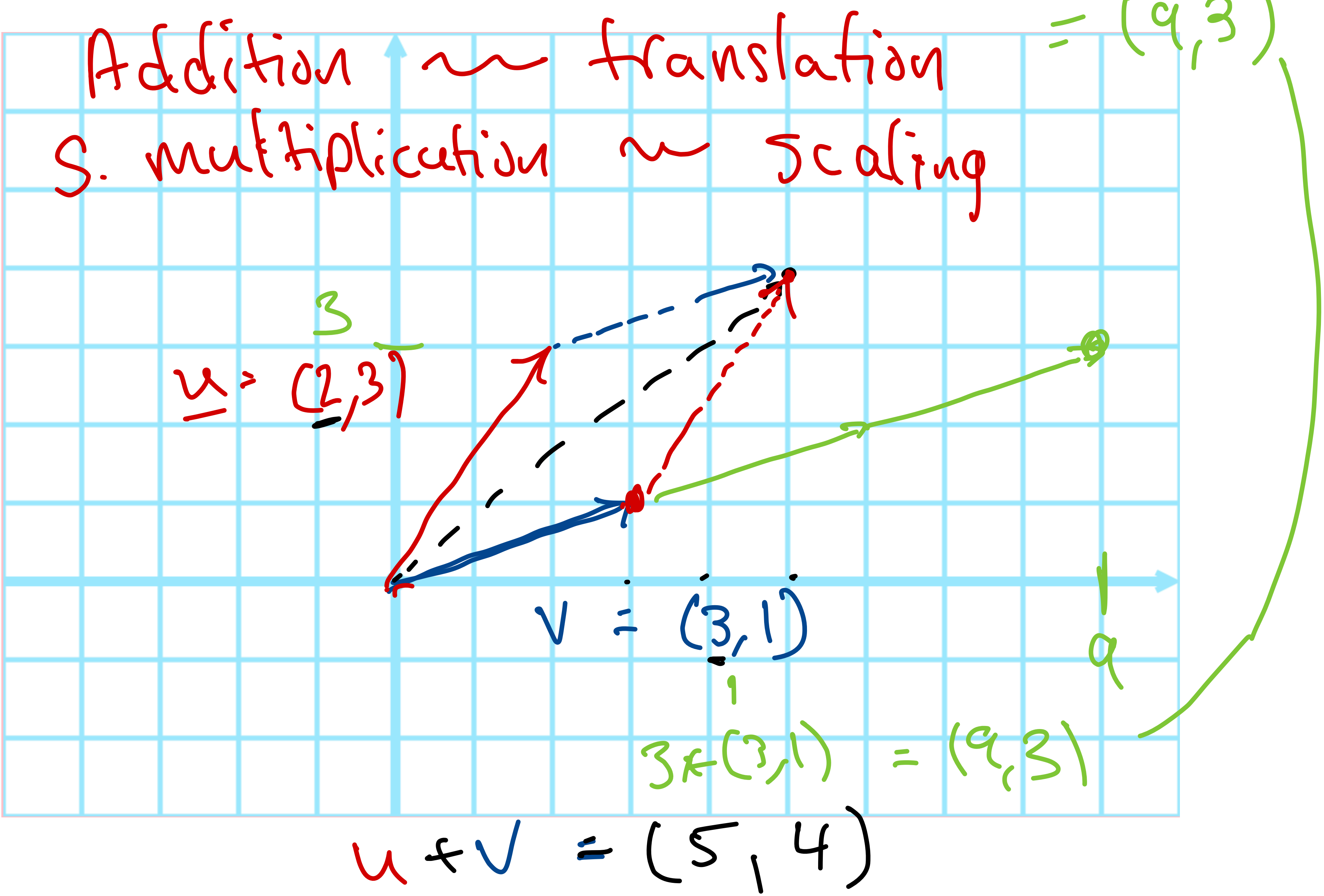
1. vector addition:
   - $(u_1, u_2) + (v_1, v_2) = (u_1 + v_1, u_2 + v_2)$
2. scalar multiplication:
   - $c(u_1, u_2) = (cu_1, cu_2)$

   scalar i.e. #

Coordinates can be interpreted as vectors:

- associate the point $A = (x, y)$ with the vector (x, y)

Vector Operations Illustrated



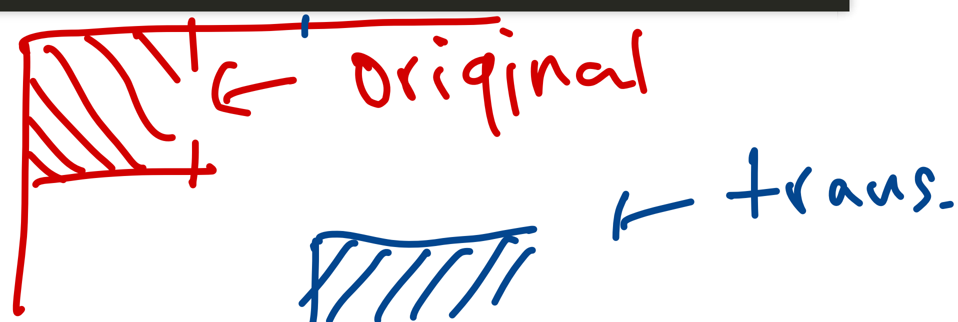
Basic Coordinate Transformations

SVG supports transformation of elements (shapes, groups, etc)

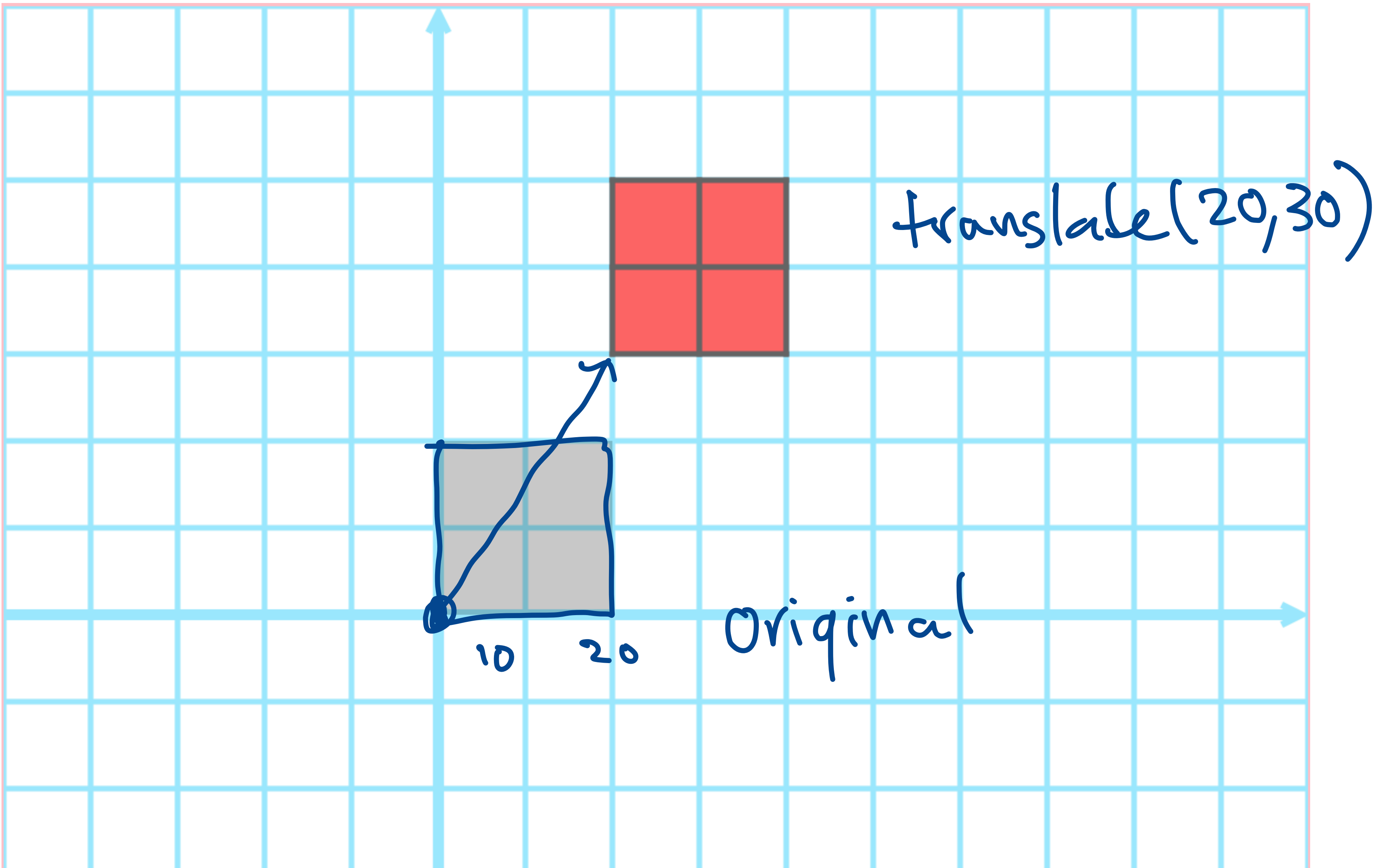
- `translate(tx, ty)`: take each vector (a, b) and move it to $(a + t_x, b + t_y)$
- `scale(s)`: take each vector (a, b) and move it to $(s \cdot a, s \cdot b)$
- `rotation(d)`: rotate each vector by d degrees in the counter clockwise direction around the origin

For example:

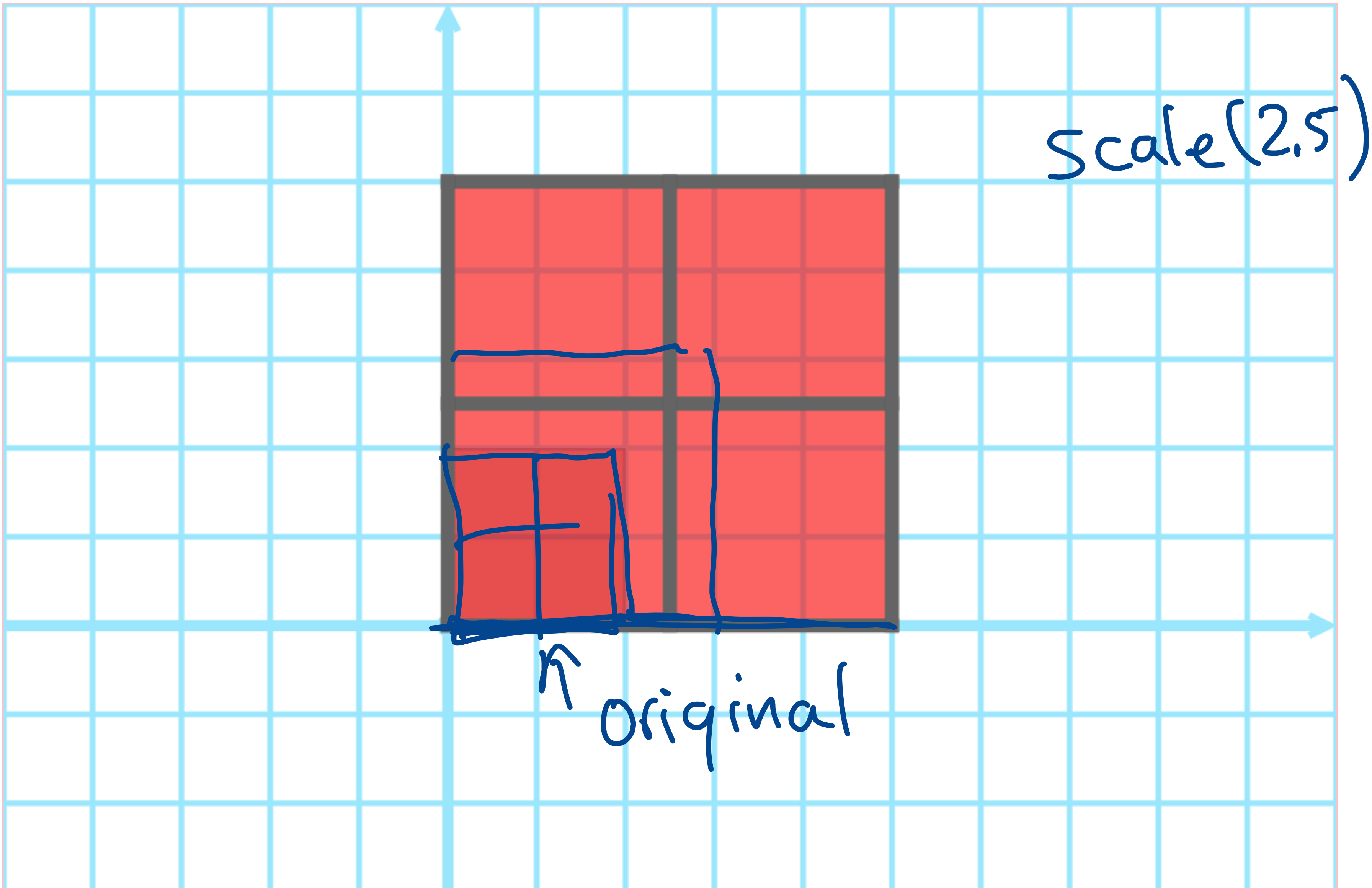
```
<rect width="20" height="20" transform="translate(30, 40)"/>
```



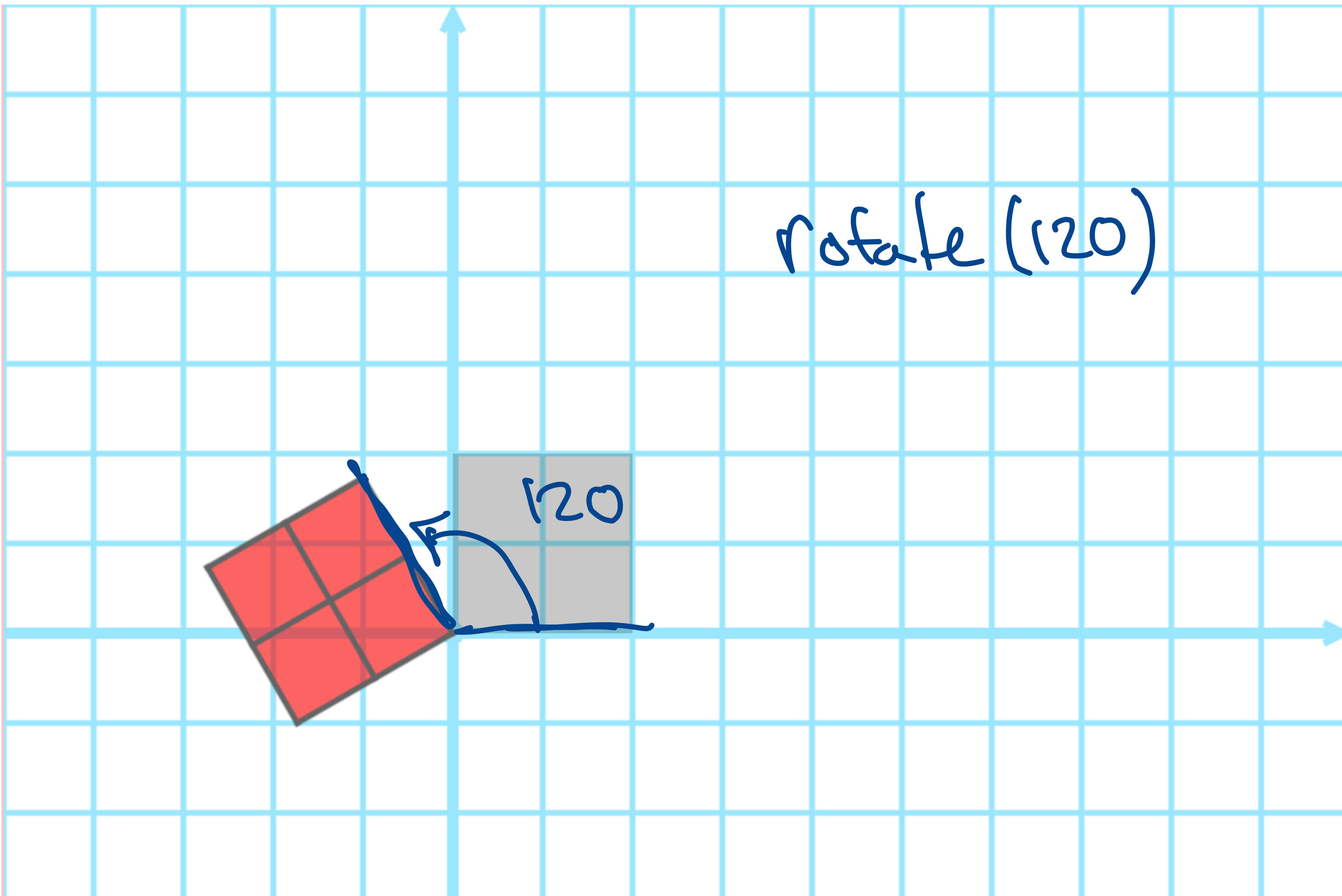
Translation



Scale



Rotation

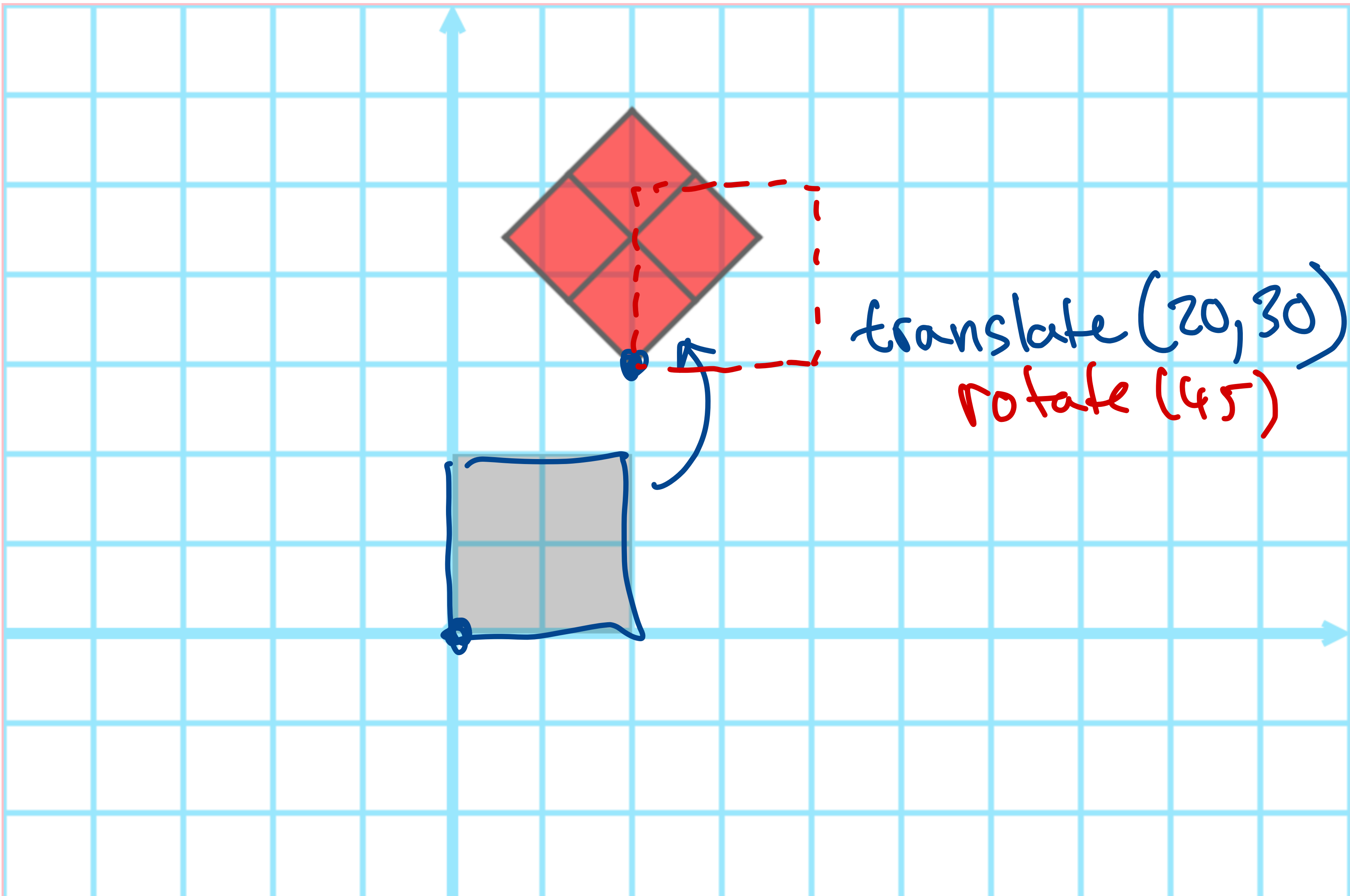


Composing Transformations

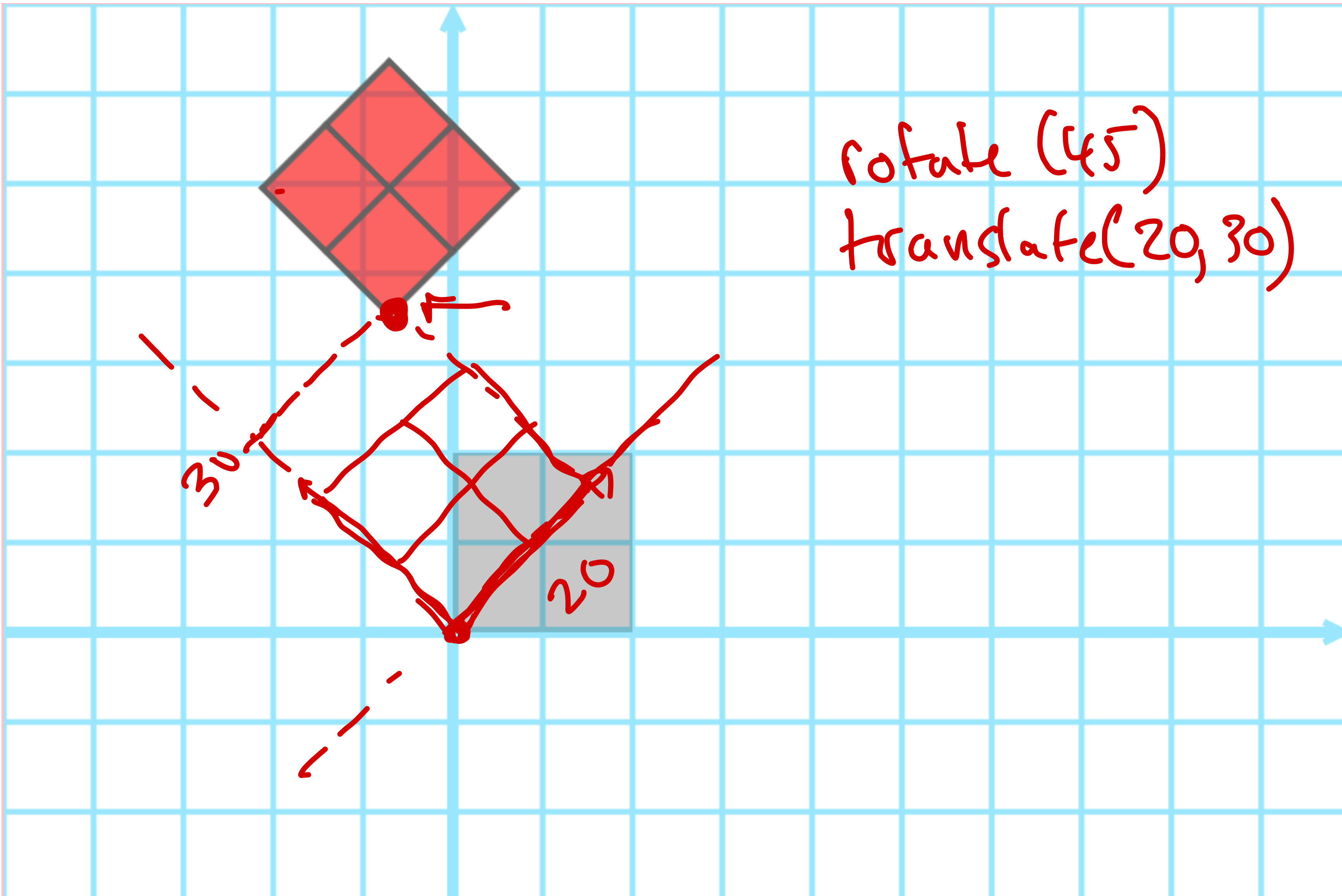
Transformations can be **composed**:

- perform one transformation, then another, ...
- transformations are applied
  - in order “left to right”
  - relative to previously transformations

Translation then Rotation

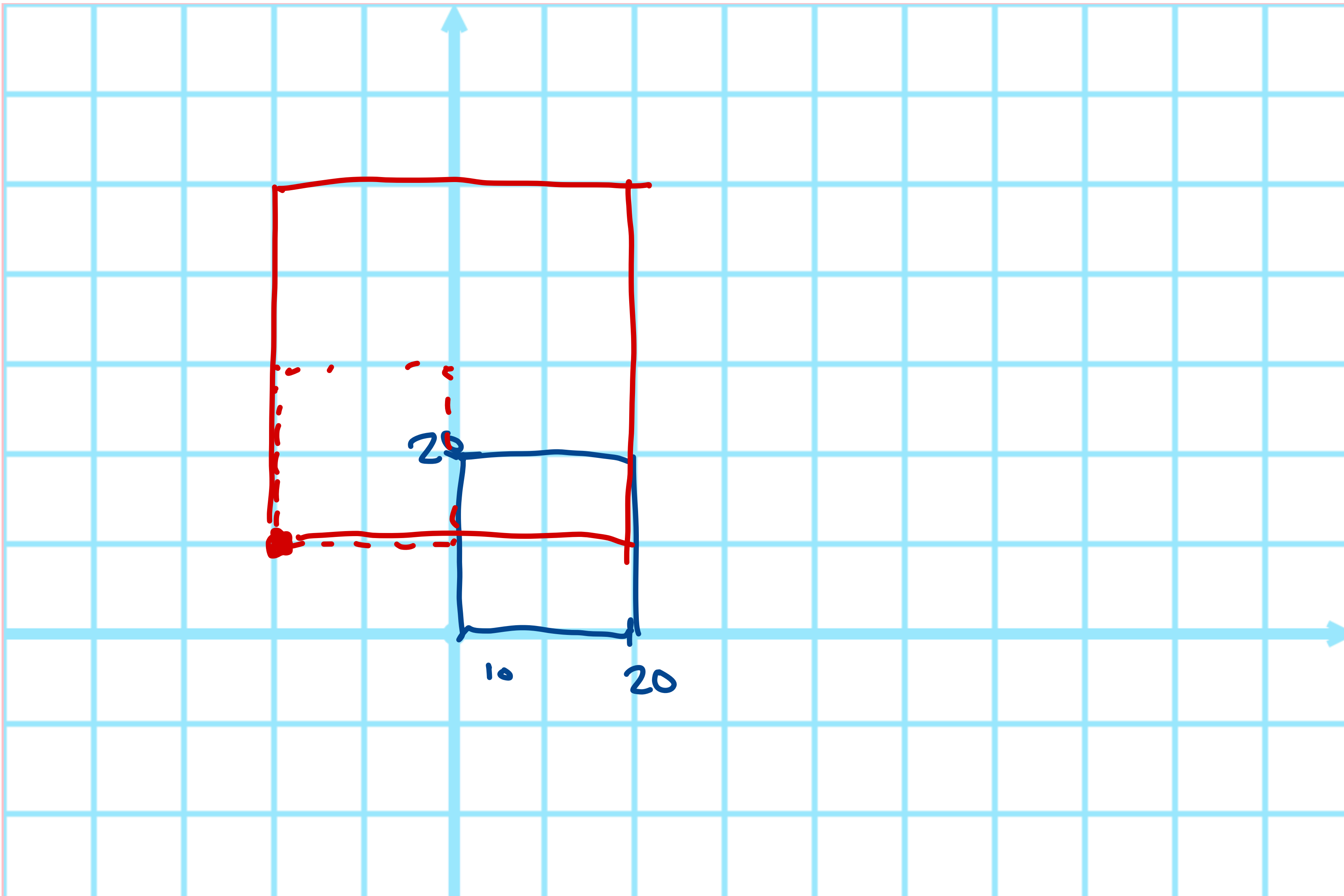


Rotation then Translation

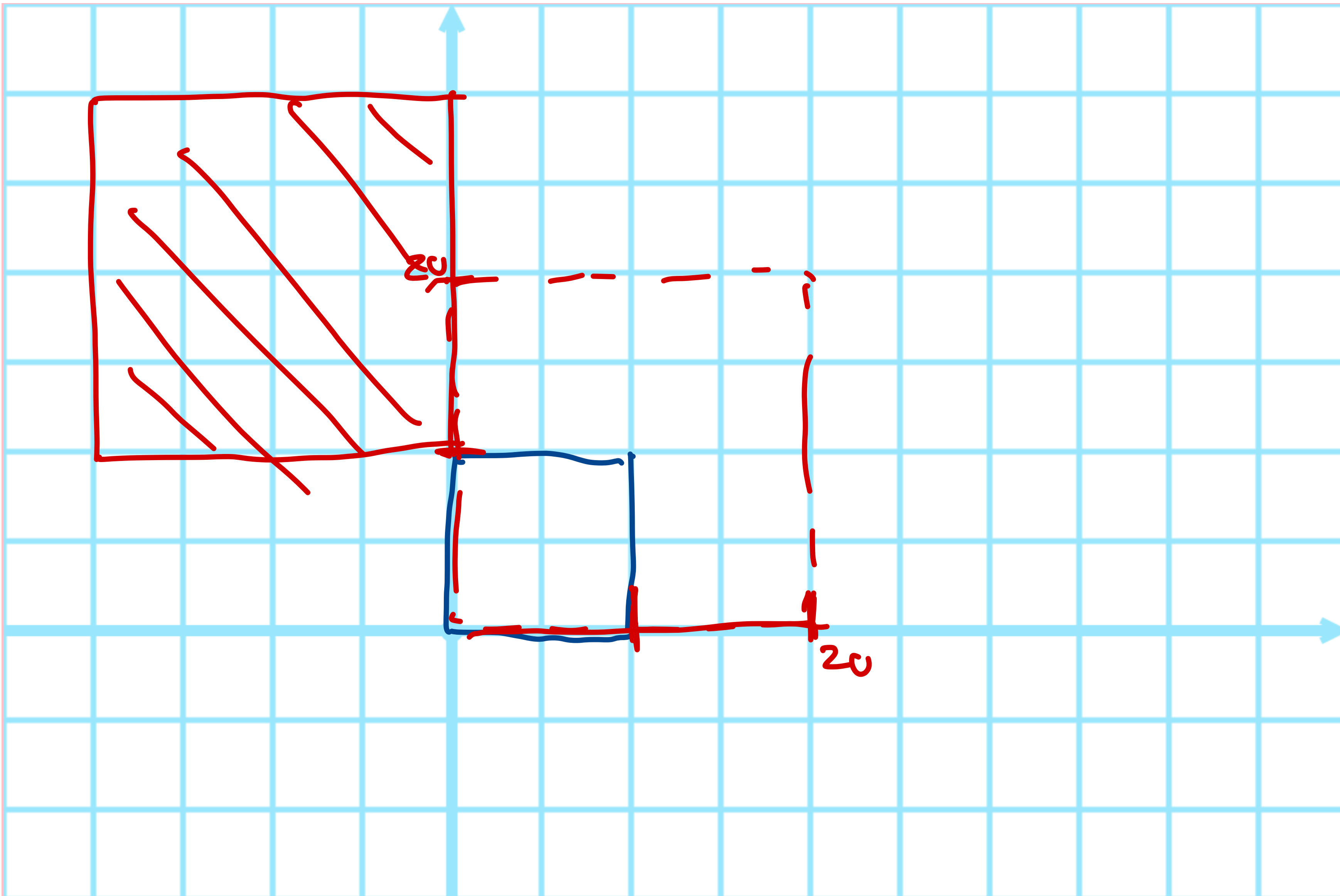


Demo

- `lec11-coordinate-transformations.zip` translate(-20, 10) scale(2)?



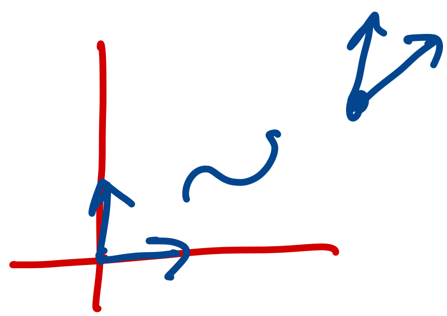
scale(2) translate(-20, 10)?



More Generally

A broad class of transformations are defined by:

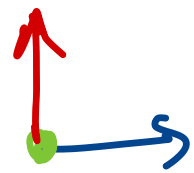
1. how they transform the **first standard basis vector** $e_1 = (1, 0)$
2. how they transform the **second standard basis vector** $e_2 = (0, 1)$
3. how they transform the **origin** $(0, 0)$

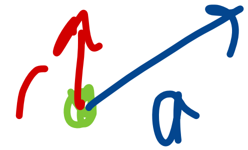


Affine Transformations

Suppose a transformation maps:

- vector $(1, 0)$ to (a, b)
- vector $(0, 1)$ to (c, d)
- point $(0, 0)$ to (e, f)





To apply transformation to (x, y):

1. write $(x, y) = x(1, 0) + y(0, 1)$
2. apply transformation to $(1, 0)$ and $(0, 1)$
3. get resulting value: $x(a, b) + y(c, d) = (ax + cy, bx + dy)$
4. add (e, f) to result:

$$(ax + cy + e, bx + dy + f)$$

This is an **affine transformation**

`matrix` Transformations

In SVG you can perform an affine transformation

- vector $(1, 0)$ to (a, b)
- vector $(0, 1)$ to (c, d)
- point $(0, 0)$ to (e, f)

with

```
transform=matrix(a, b, c, d, e, f)
```

`matrix` transforms include all `scale`, `translate`, `rotate` transforms, and more!

Questions

1. What is the `matrix` equivalent of `translate(20, 30)`?

   matrix(1, 0, 0, 1, 20, 30)

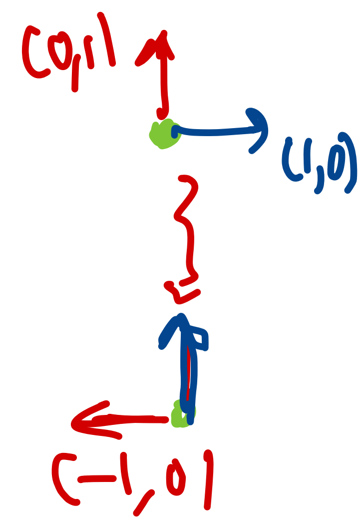
2. What is the `matrix` equivalent of `scale(2)`?

   matrix(2 0 0 2 0 0)

3. What is the `matrix` equivalent of `rotate(90)`?

   matrix(0 1 -1 0 0, 0)

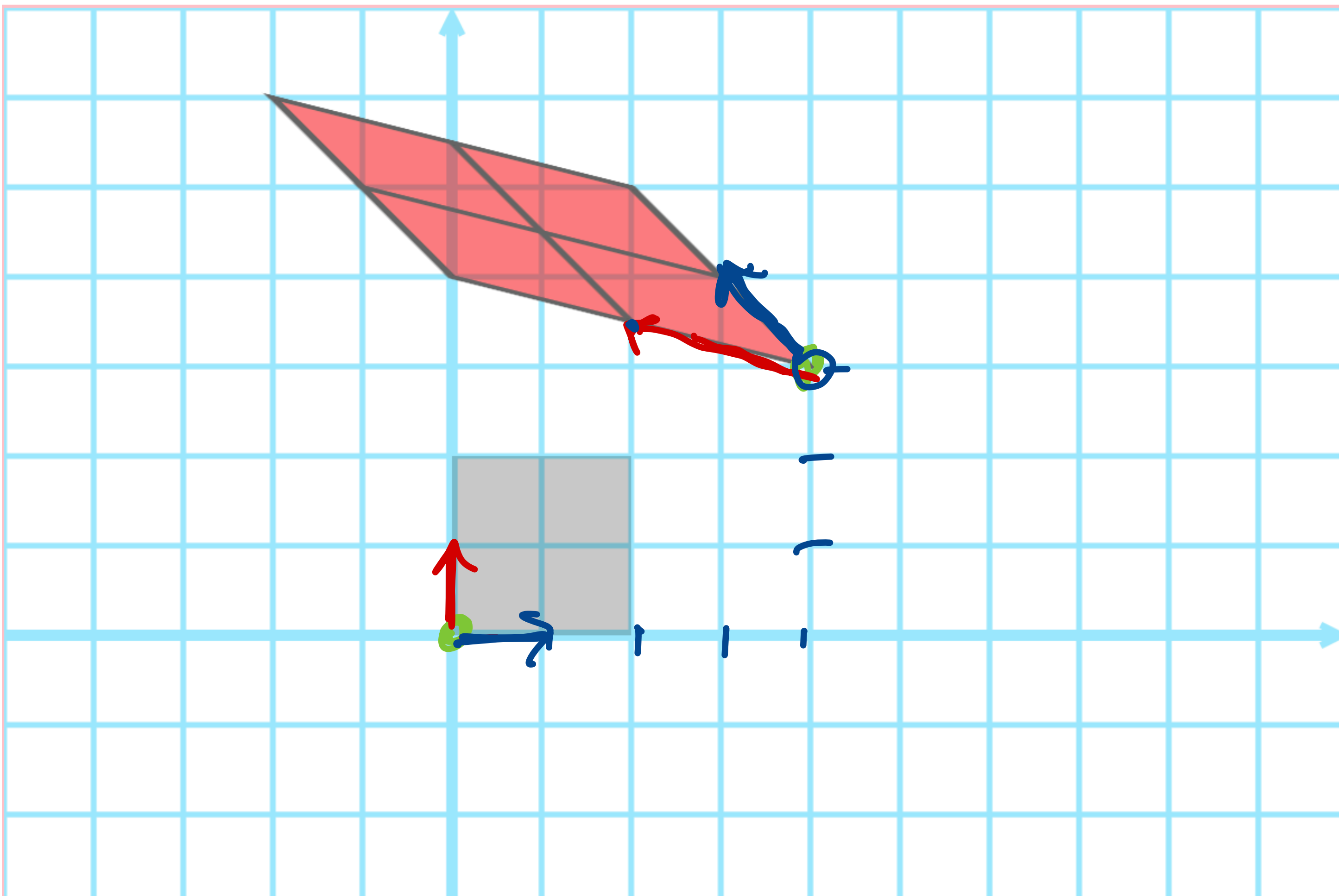
   cos θ sin θ -sin θ cos θ



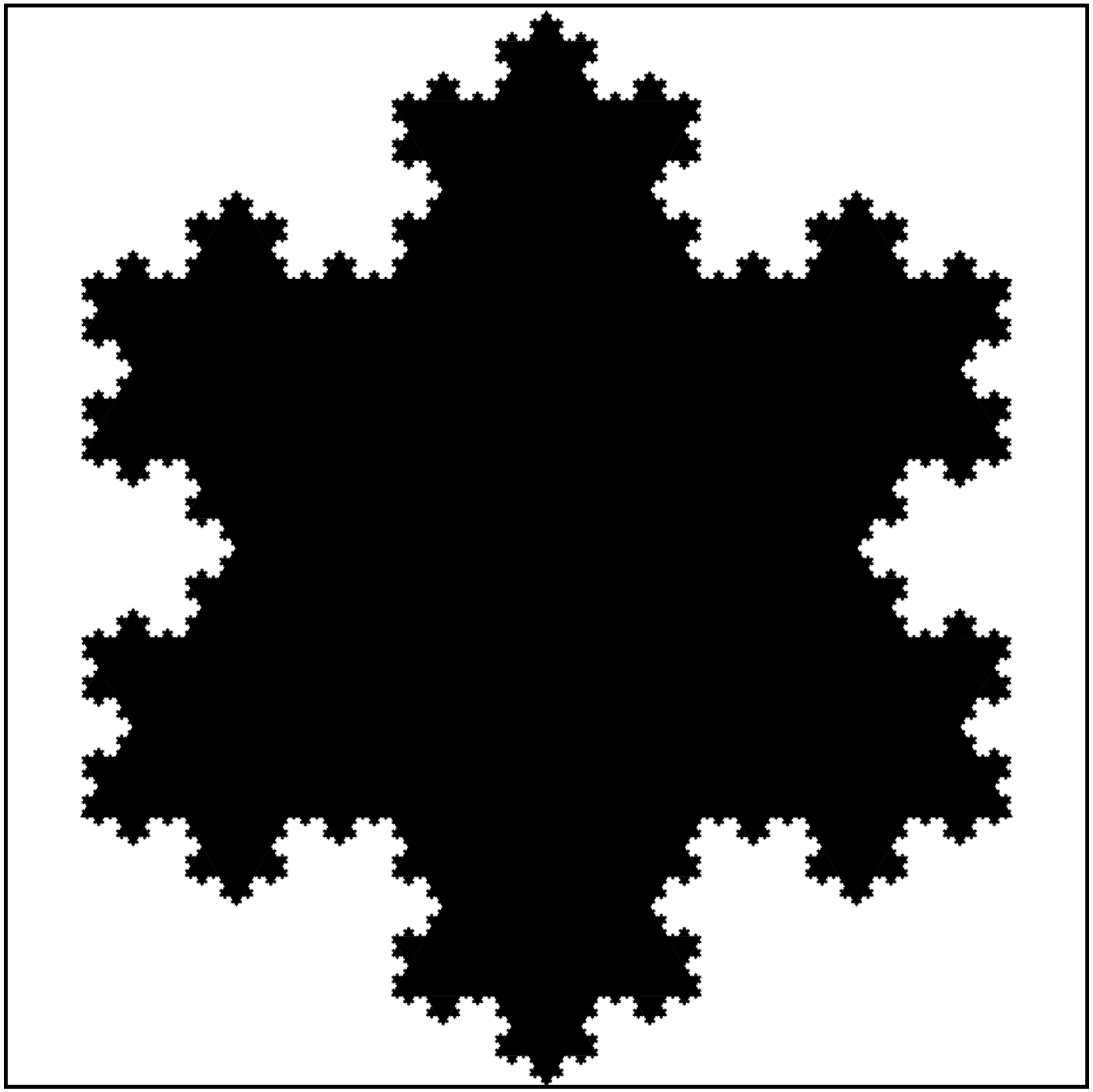
Question

matrix(-1 1 -2 0.5 4 0 3 0)

What is the matrix of this transformation?

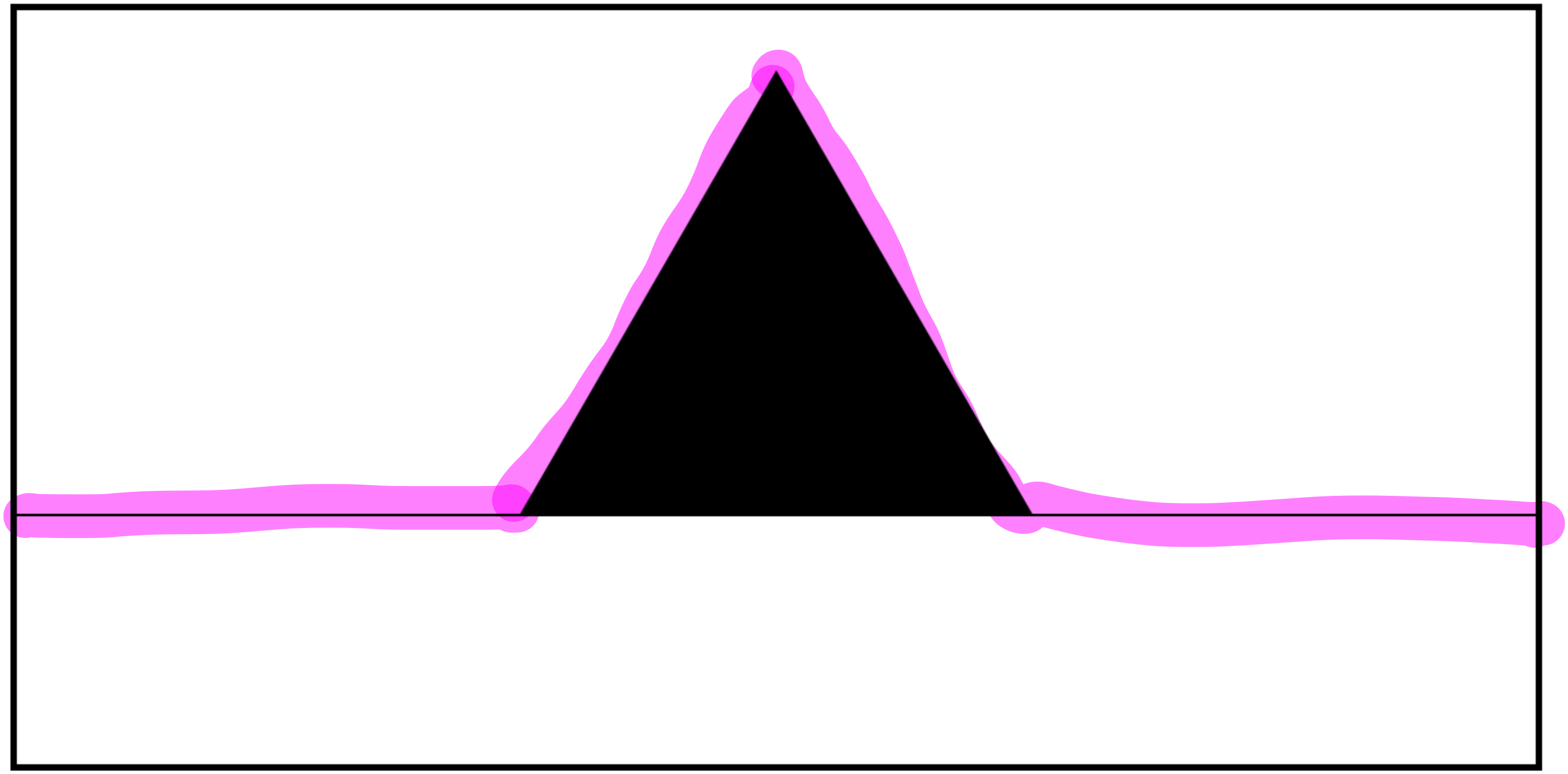


Self-Similarity via Transformations



Example: Koch Curve I

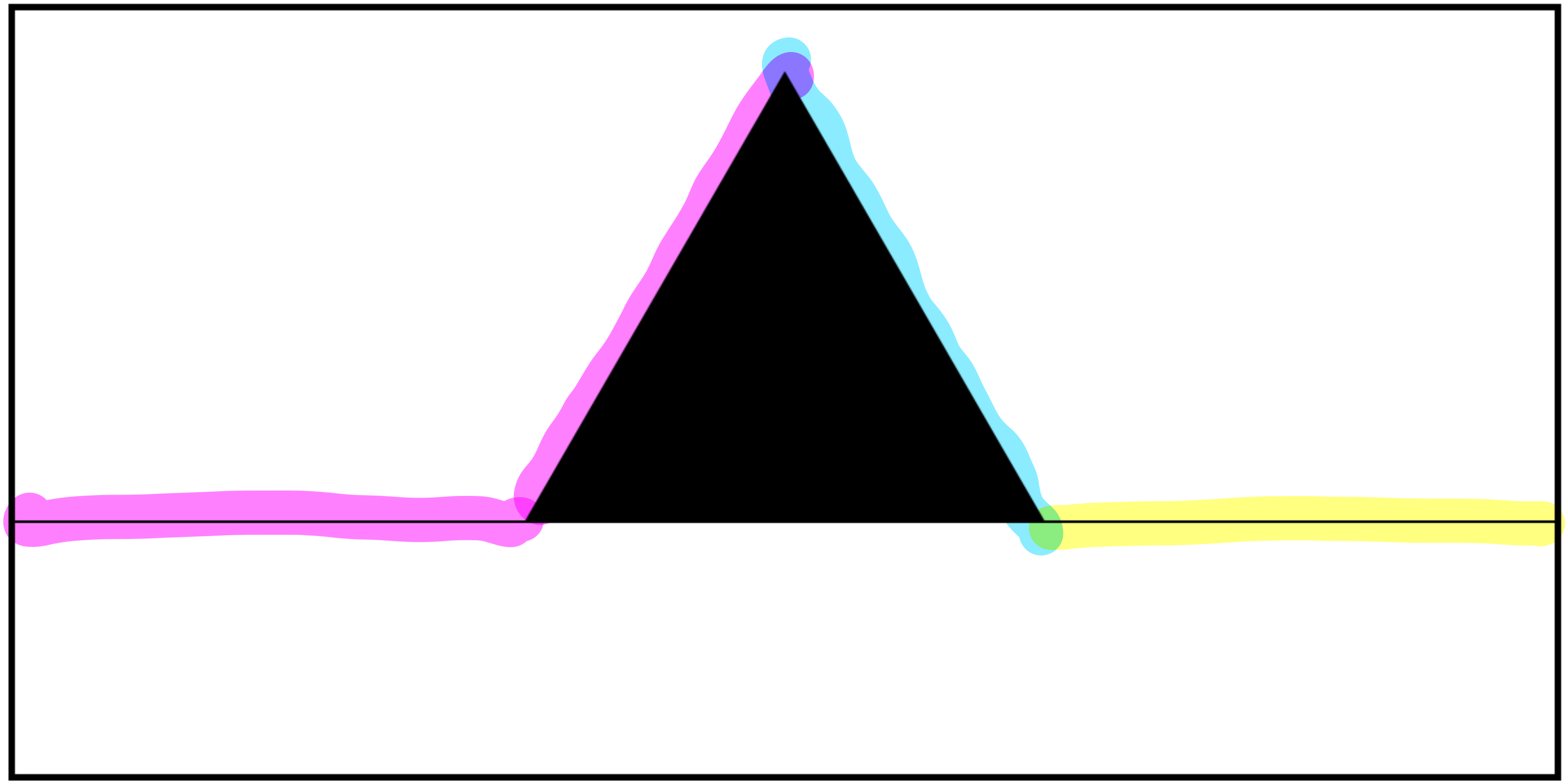
How did we make the snowflake fractal?



Step 1: define a basic shape

Example: Koch Curve II

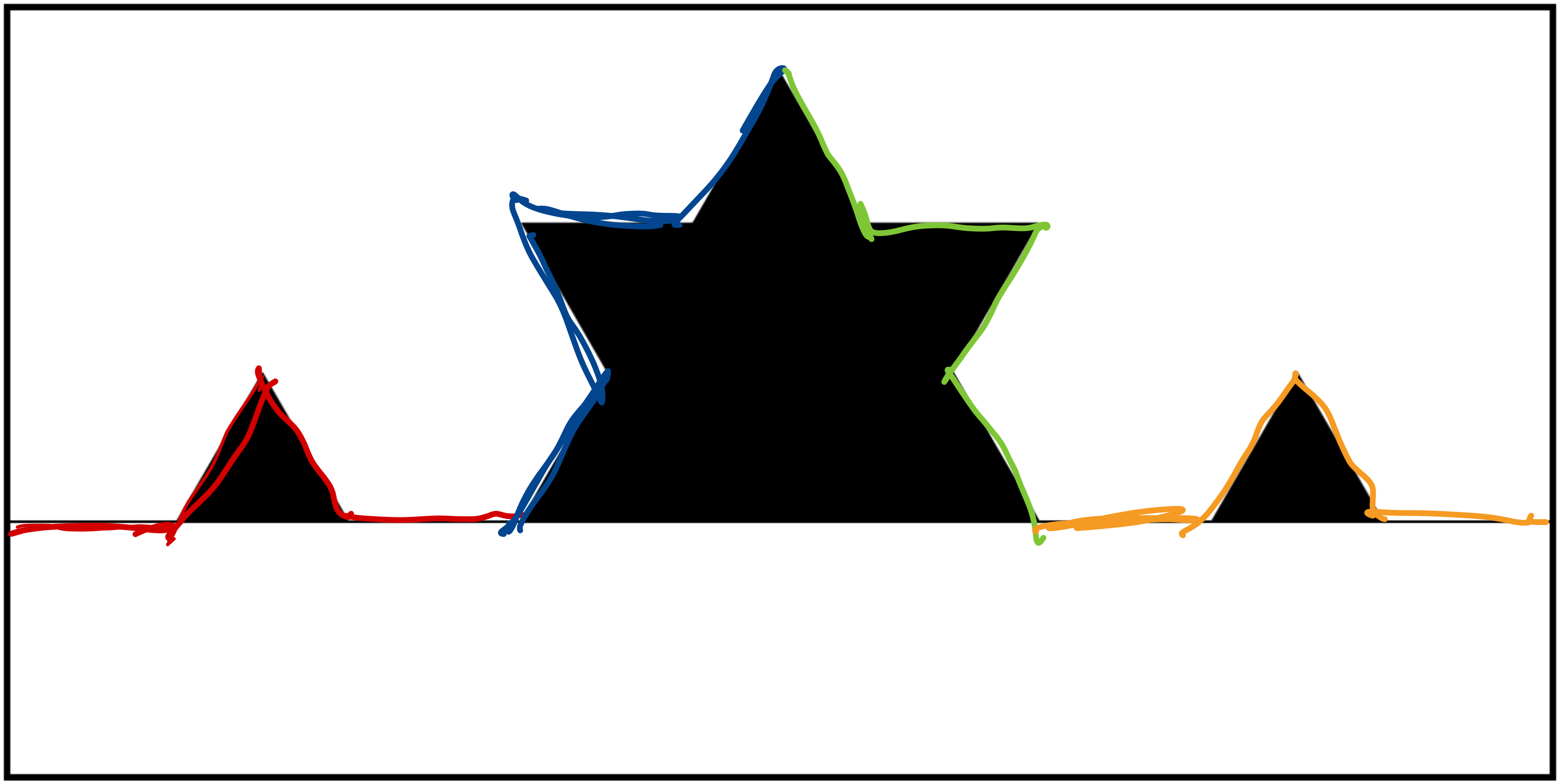
How did we make the snowflake fractal?



Step 2: define sub-shapes for basic shape

Example: Koch Curve III

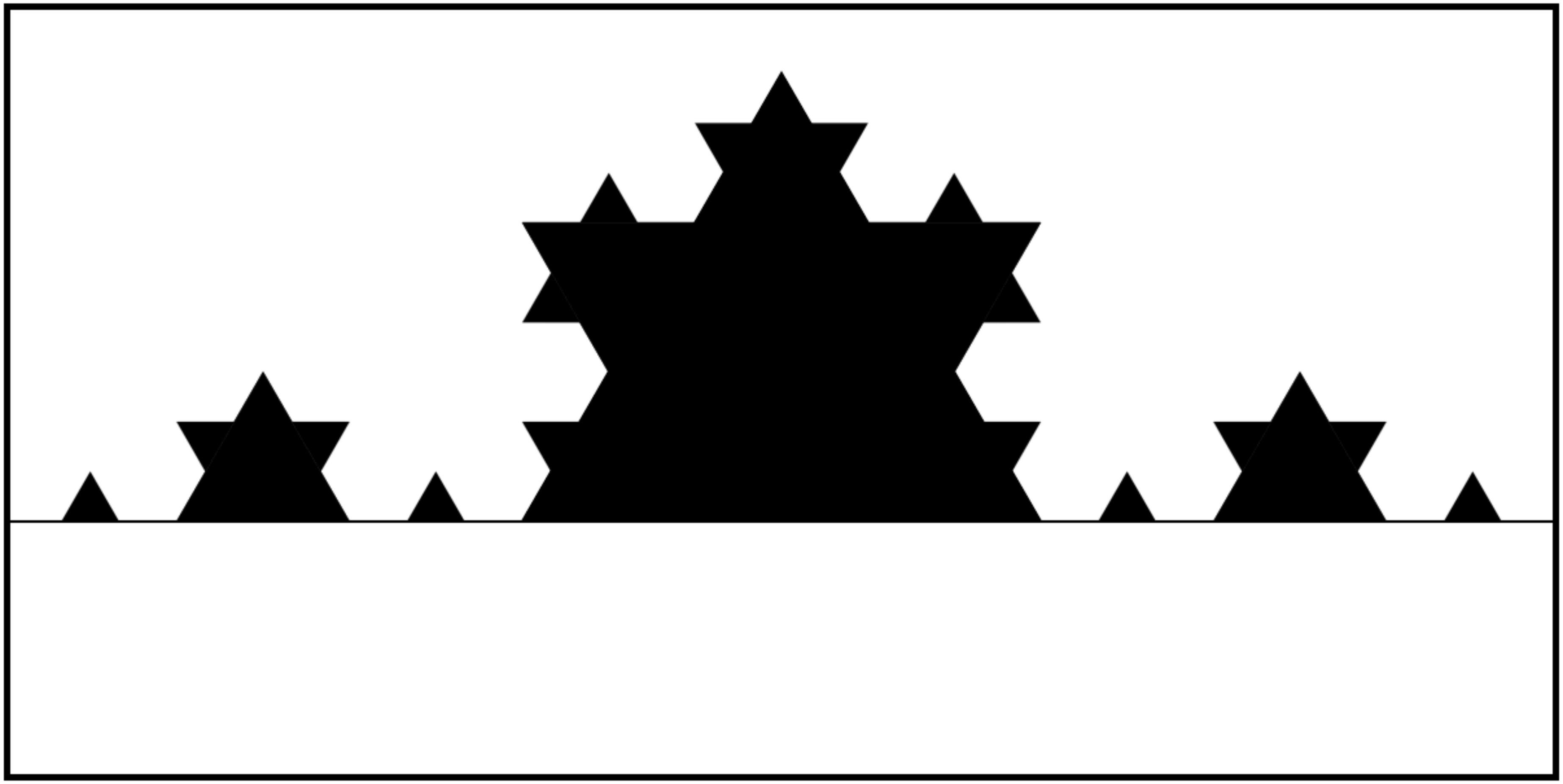
How did we make the snowflake fractal?



Step 3: recurse

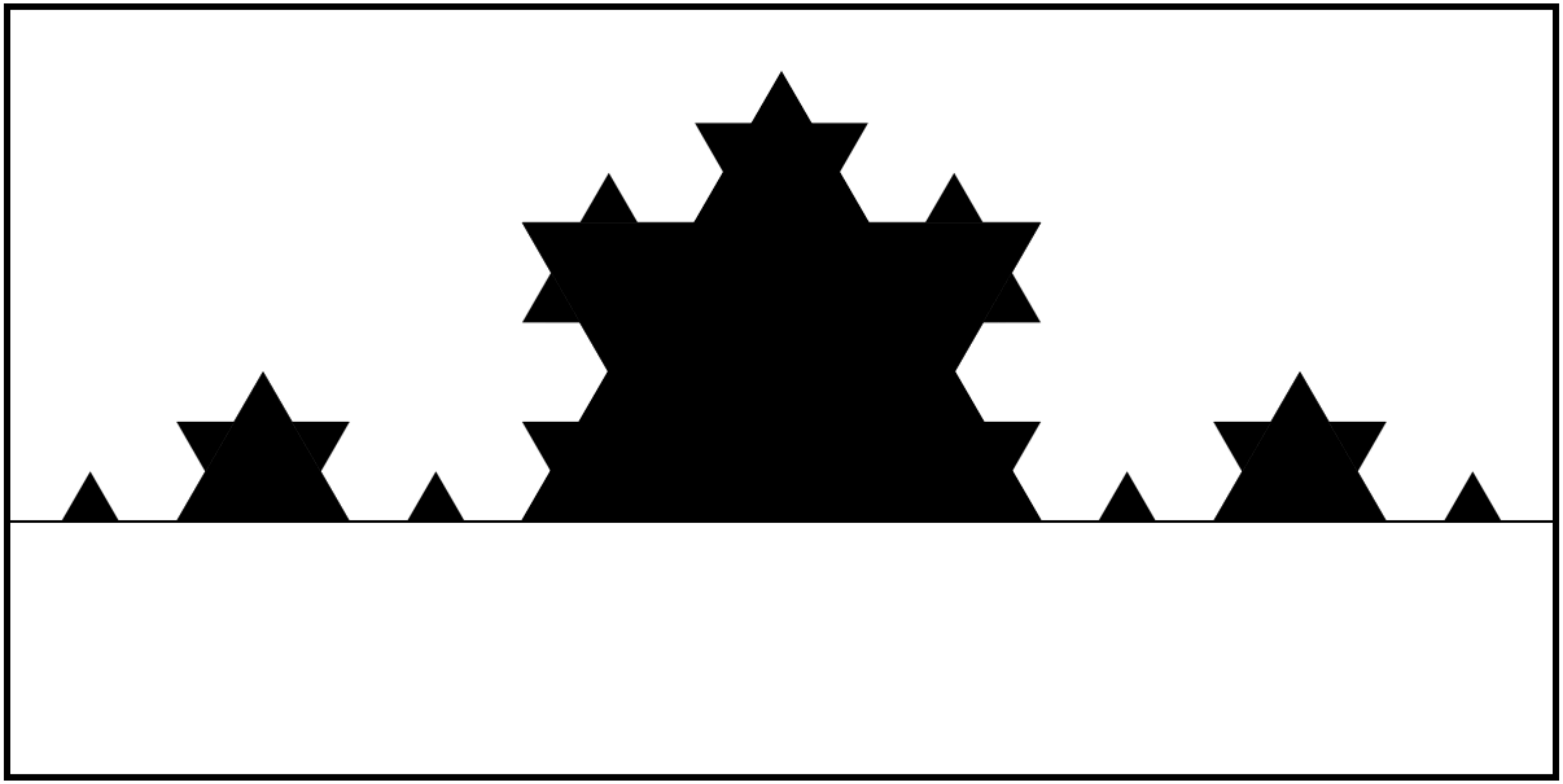
Example: Koch Curve IV

How did we make the snowflake fractal?



Step 3: recurse

Observation



Each iteration draws a bunch of *transformed* copies of the original shape

Repetition and Transformation in SVG

1. Define the basic shape in a `<defs>` element

```
<defs>
    <rect id="my-rect" width="20" height="20"/>
</defs>
```

2. Draw basic shape with `<use>`, apply `transform` to transform the element

```
<use href="#my-rect" transform="translate(20, 30)"/>
```

Now can re-use `my-rect` over and over again with different transformations

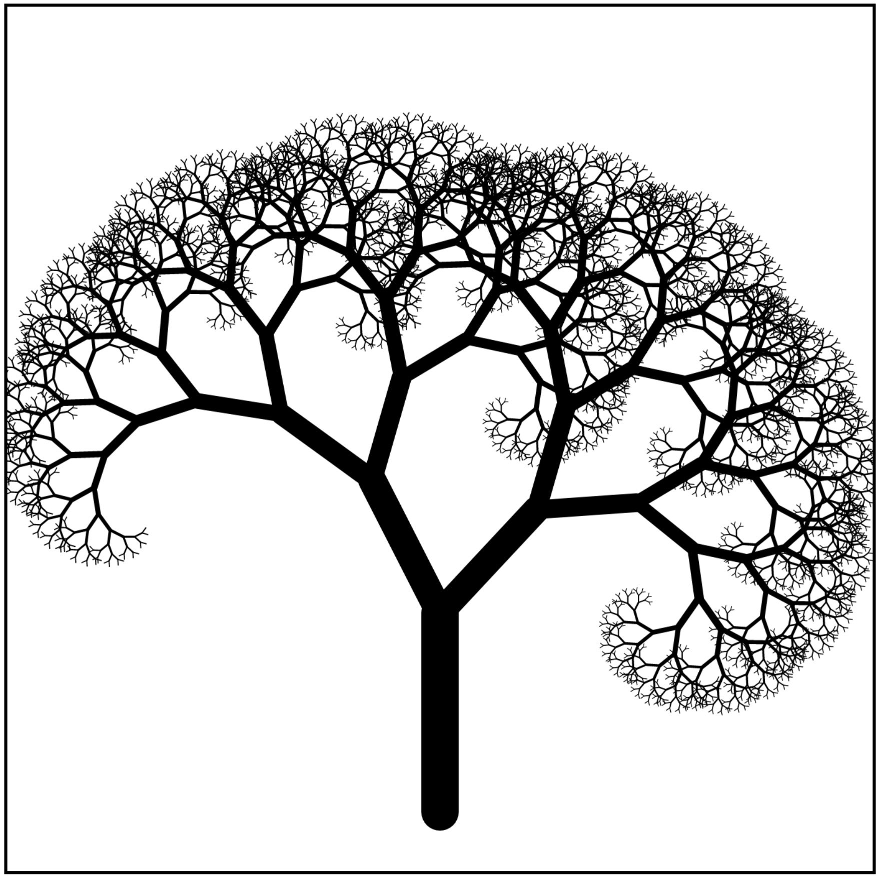
- of course, this can (should?) all be done with JavaScript

Activity

Draw two iterations of the Koch curve!

- `lec11-koch-step.zip`

Next Time



Make things easier!

- compose transformations by nesting group (<g>) elements

- program drawing recursively