

## Quiz 03

COSC 373: *Distributed Algorithms*, Spring 2022

**Instructions.** This quiz is open book and open note—you may freely use your notes, lecture notes, or textbook while working on it. You may *not* consult any living resources such as other students or web forums. The quiz should be submitted through Gradescope by 11:59pm on Friday, April 22nd.

In answering the questions on the quiz, you should provide a high-level justification of your solution—it is *not* expected that you provide a formal proof of your solutions, or that you argue directly from first principles. Your solutions may use any algorithm from class as a subroutine, and your justification can quote any result described in class or previous homework assignments.

**Affirmation.** I attest that that work presented here is mine and mine alone. I have not consulted any disallowed resources while taking this quiz.

Name: \_\_\_\_\_

Signature: \_\_\_\_\_

**Question 1.** Recall that a *proper  $k$  coloring* of a graph  $G = (V, E)$  assigns to each vertex  $v \in V$  a *color* from the range  $\{1, 2, \dots, k\}$  such that adjacent (neighboring) vertices are always assigned distinct colors. Suppose each node  $v \in V$  receives as input  $\Delta$  the maximum degree of any node in the network, and a color  $c = c(v)$  such that the input colors form a proper  $k$ -coloring of  $G$ . Devise an algorithm in the Port Ordering (PO) model that computes a maximal matching on  $G$  in  $O(k\Delta)$  rounds.

**Question 2.** Consider the *distinct inputs* problem, DI defined as follows. Each node  $v$  receives as input a number  $N(v)$  from the range  $1, 2, \dots, 2n$  where  $n$  is the size (number of nodes) of the network. All nodes should output 1 if all inputs are pairwise distinct, and 0 otherwise. That is, if any two nodes in the network receive the same input, then all nodes should 0. Use an indistinguishability argument to show that in the LOCAL model, any algorithm that solves DI requires at least  $D$  rounds on every graph with diameter  $D$ .

**Question 3.** The problem N\_EDGES requires each node in the network to output  $m$ , the number of edges in the network. Devise an algorithm in the CONGEST model that solves N\_EDGES in  $O(D)$  rounds, where  $D$  is the network diameter. (*Hint: What is the connection between the degrees of the nodes and the number of edges in the graph?*)