Homework 4

COSC 373, Spring 2022

Will Rosenbaum
Updated: April 8, 2022

Department of Computer Science
Amherst College

Question 1. Suppose G is a graph with n vertices and maximum degree Δ. Recall that a ***proper*** k ***coloring*** of G is an assignment of a "color" $c(v)$ from the range $1, 2, \ldots, k$ to each vertex v such that neighboring vertices are always assigned different colors. Consider the following coloring procedure:

Repeat until done:

1. each node v picks a uniformly random color $c(v)$ from the range $1, 2, \ldots, 2\deg(v)$
2. if $c(v) \neq c(u)$ for all neighbors u of v, v outputs $c(v)$ and halts

Express this procedure as a protocol in the CONGEST model, and show that it produces a proper 2Δ coloring of G in $O(\log n)$ rounds with high probability. *(Hint: what is the probability that a given node halts in a given round?)*

Question 2. Suppose A and B are subsets of $[N] = \{1, 2, \ldots, N\}$. In class we considered the ***disjointness function***, $\mathrm{DISJ}(A, B)$ defined by

$$\mathrm{DISJ}(A, B) = \begin{cases} 1 & \text{if } A \text{ and } B \text{ are disjoint} \\ 0 & \text{otherwise.} \end{cases}$$

Given a set $A \subseteq \{1, 2, \ldots, N\}$, the ***complement*** of A, denoted A^c, is the set of elements from $\{1, 2, \ldots, N\}$ that are *not* contained in A. Let

$$F = \{(A, A^c) \mid A \subseteq \{1, 2, \ldots, N\}\}.$$

That is, F is the set of all pairs of sets (A, B) where B is the complement of A. Show that F is a fooling set for DISJ.

Question 3. Suppose $x, y \in \{0, 1\}^N$ are N-bit strings, which we interpret as (binary representations of) integers. That is, we can view x and y as numbers between 0 and $2^N - 1$. The ***greater than*** function, GT, is defined to be

$$\mathrm{GT}(x, y) = \begin{cases} 1 & \text{if } x > y \\ 0 & \text{otherwise.} \end{cases}$$

Use the fooling set method to prove that $D(\mathrm{GT}) \geq N$. That is, the deterministic communication complexity of GT is at least N.