Principle of Induction. Suppose $P(1), P(2), P(3), \ldots$ is a sequence of predicates. Suppose we establish that (1) $P(1)$ is true (*base case*), and (2) for all i, if $P(i)$ is true, then $P(i+1)$ is also true (*inductive step*). Then for every n, $P(n)$ is true.

Properties of O. You freely assume and use the following properties of O notation:

1. If $f(n) \leq a$ for all n, then $f = O(1)$.
2. If $f(n) \leq g(n)$ for all n, then $f = O(g)$.
3. If $f = O(g)$, then $a \cdot f = O(g)$.
4. If $f = O(g)$ and $g = O(h)$, then $f = O(h)$.
5. If $f = O(g)$, then $f + O(g) = O(g)$ and $g + O(f) = O(g)$ (in particular $f + g = O(g)$)
6. If $f_1 = O(g_1)$ and $f_2 = O(g_2)$, then $f_1 \cdot f_2 = O(g_1 \cdot g_2)$.
7. If $a \leq b$ then $n^a = O(n^b)$
8. If $a < b$ then $n^b \neq O(n^a)$
9. For every constant $c > 0$, $\log n = O(n^c)$
10. For all constants $c \geq 0$ and $a > 1$, $n^c = O(a^n)$
11. If $f = O(g)$ then $g = \Omega(f)$
12. If $f = O(g)$ and $f = \Omega(g)$, then $f = \Theta(g)$.

Master Theorem. Suppose the function T satisfies the recurrence relation:

$$T(n) = aT(n/b) + f(n).$$

Define $c = \log_b a$. Then the following hold:

1. If $f(n) = O(n^d)$ for $d < c$ then $T(n) = O(n^c)$
2. If $f(n) = \Theta(n^c \log^k n)$ then $T(n) = O(n^c \log^{k+1} n)$
3. If $f(n) = \Omega(n^d)$ for $d > c$, then $T(n) = O(f(n))$